Information Sciences 385–386 (2017) 1–18

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Multi-period portfolio selection with dynamic risk/expected-return level under fuzzy random uncertainty

Bo Wang[a,*], You Li[b], Junzo Watada[c]

[a] School of Management and Engineering, Nanjing University, Nanjing 210093, China
[b] School of Finance, Nanjing University of Finance and Economics, Nanjing 210046, China
[c] Graduate School of Information, Production and Systems, Waseda University, Kitakyushu 808-0135, Japan

ARTICLE INFO

Article history:
Received 30 November 2015
Revised 24 December 2016
Accepted 26 December 2016
Available online 27 December 2016

Keywords:
Multi-period portfolio selection
Dynamic risk/expected-return level
Fuzzy random variables
Particle swarm optimization

ABSTRACT

In this study, we discuss multi-period portfolio selection problems when security returns are described as fuzzy random variables. The main concern of this work is to apply dynamic risk tolerance and expected return levels in mathematical modeling; *i.e.*, these two indices of each period are influenced by the investment result of the previous period as well as human risk attitudes instead of static values over the entire investment horizon. Essentially, this assumption is based on the reality that investors tend to update targets when their wealth changes. In addition, fuzzy random variables are employed here to incorporate historical data with expert knowledge when estimating security future returns. Based on the above considerations, two multi-period portfolio selection models are built in light of the different risk attitudes. We then provide property analysis on complicated nonlinear optimization problems and derive several equivalents of the models, which can be solved by the existing dynamic programming. In general situations, a fuzzy random simulation-based particle swarm optimization algorithm is developed to search for approximate optima. The performance of this research is exemplified by a real market data-based case study in which the superiority of the dynamic strategy is demonstrated by a comparison with conventional approaches.

1. Introduction

Portfolio selection pursues optimal capital allocations among a basket of securities, so that investors' preferences on return and risk can be satisfied. A pioneer in the field, Markowitz [32] developed the classical mean-variance model that establishes the single period variance of security returns as a risk measurement. Since then, various studies have been proposed that improved modern portfolio selection theory from two perspectives: approaches to security return uncertainty and criterion for estimating portfolio risk.

Most studies treat security returns as random variables stem from stochastic analysis on precise historical data. This is reasonable as data knowledge does reflect certain aspects of security returns. Nevertheless, due to the complexity of the stock market, security returns in general are influenced by manifold factors such as company performance, market forces of supply and demand, positive/negative news and political issues, which jointly form the inputs of return forecasts [39]. On one hand, historical data contains little information regarding the above inputs. On the other hand, these influence factors

* Corresponding author.
E-mail addresses: bowangsme@nju.edu.cn (B. Wang), ruby1025@gmail.com (Y. Li), junzow@osb.att.ne.jp (J. Watada).

http://dx.doi.org/10.1016/j.ins.2016.12.033

are often assessed with some level of ambiguity, as they are typically non-statistical due to practical difficulties of data acquisition, the nature of measurement imprecision and the vagueness of human perception, a joint fact in many real-world applications. Therefore, researchers have argued that randomness is not the only type of uncertainty in reality [10,15,23] and expert knowledge could be invoked to enhance forecast accuracy [16,39]. Recently, fuzzy set theory [47] has been applied as a widely acceptable tool to deal with expert knowledge as well as the non-statistical uncertainty involved in security return forecasts. It is noteworthy that one of the significant characteristics of fuzzy set theory lies in its linguistic elasticity, which accounts for ambiguities in an uncertain environment without being restricted by detailed level information [20,41]. Considering this merit, a number of fuzzy portfolio selection models have been proposed in recent years when the security returns were described as fuzzy variables [3,9,14,17,33,37,44].

The above knowledge shows that security return uncertainty generally consists of two types *i.e*., random uncertainty and fuzzy uncertainty, which are often mixed up with each other in real-life applications [12]. For random uncertainty, the probabilistic distribution function of security returns could be partially realized from stochastic analysis on historical data. Nevertheless, the security returns of each random event may not be exactly captured and are often assumed to be fuzzy variables due to the non-statistical feature of expert knowledge as well as the various influence factors mentioned above. Thus, investors are faced with "random returns with fuzzy information" [12]. Considering that a fuzzy random variable is a measurable function from a probability space to a collection of fuzzy sets, it could be reasonable and imperative to apply fuzzy random theory when dealing with hybrid uncertainty in portfolio optimization [34]. In the literature, Hao and Liu [12] developed two portfolio selection models where the security returns are characterized by fuzzy random variables with known possibility and probability distributions. A genetic algorithm (GA) was then employed to solve the problems. Nguyen et al. [34] described security returns as LR fuzzy random variables, whereby a reward-to-uncertainty ratio is proposed together with the Sharpe ratio to build the portfolio selection model. Li et al. [25] employed technical analysis to construct a fuzzy random portfolio selection model, and an improved particle swarm optimization (PSO) algorithm was designed as the solution. In view of return, risk and liquidity, Li et al. [24] proposed a constrained multi-objective portfolio selection model with fuzzy random returns; it was solved by a compromise approach-based GA. To incorporate data information with expert knowledge properly, the imprecise security returns in this study are also characterized by fuzzy random variables. The detailed knowledge as well as the method advantages are illustrated in Section 3.

In addition to uncertainty representation, risk measurements also play pivotal roles in modern portfolio selection theory. Inspired by the distinguished mean-variance model, researchers have developed various techniques in either a stochastic or fuzzy environment to evaluate portfolio risks from different perspectives, *e.g*., chance-constraints [2], mean-semivariance [13], mean-variance-skewness [38], mean-entropy [14], Value-at-Risk (VaR) [18,39] and conditional VaR [35,46]. Basically, these measurements can be divided into two categories: One is to reduce investment risks by inspiring the diversification of capital allocations and includes most existing approaches such as mean-variance and mean-entropy. The second measures risk in terms of the exact loss, *e.g*., VaR and conditional VaR, which show the largest loss of an investment under a given confidence level. The literature as well as experience demonstrate that the second type of risk measurement is more acceptable for general investors. The reason is twofold: First, variance or entropy provides little information about how much wealth investors may lose, while it is the loss of money that concerns investors the most [16]. Therefore, compared with VaR, these techniques are less sensitive to investors and thus may introduce salient difficulties in assigning risk tolerance levels or estimating portfolio performance. Second, VaR or conditional VaR provides robust evaluation on investment risks, as a result producing low-risk portfolio decisions for general investors, in particular for risk-averters. Based on the above features, we adopt fuzzy random VaR as the risk measurement in subsequent modeling.

All of the aforementioned studies treat portfolio selection as single-period problems, *i.e*. the allocation decision made at the beginning is assumed to be static until the end of an investment. Nevertheless, this hypothesis is more or less contrary to investors' intuition, as they always vary portfolio with time. Consequently, it is natural to extend the above works to multi-period portfolio selection. In an early study, Li and Ng [22] reported a multi-period mean-variance formulation to capture the spirit of risk management in a dynamic investment structure. More importantly, an analytical optimal solution and optimal portfolio policy were derived. Huang [16] investigated multi-period portfolio selection in fuzzy environments, where the security returns were described as fuzzy variables given by expert knowledge. More recently, Cui et al. [4] introduced a mean-field framework as an efficient modeling tool to directly deal with the non-separability multi-period mean-variance portfolio selection problem, rather than invoking auxiliary parametric formulations.

However, to the best of our knowledge, the existing studies take the risk/expected-return level as a constant over the investment horizon, while these values could be affected by the investment result of the previous period as well as human risk attitudes. For example, when the actual profit of period $t-1$ is greater than the investor's expectation, he is apt to be less risk averse or more risk seeking when determining the next portfolio. In other words, the risk level of period t could be relaxed as the investor holds excess wealth. On the flip side, if the return is lower than the expectation, the investor is inclined to be more risk averse. These assumptions are acceptable and follow common sense. Therefore, in this study, we apply dynamic risk tolerance and expected return levels in mathematical modeling, the detailed knowledge of which is presented in Section 3.

Being notoriously non-separable problems, the standard multi-period portfolio selection model has been reported as being ill suited to find optimal solutions. Unfortunately, the consideration of dynamic risk/expected-return level and the utilization of fuzzy random variables introduce further difficulties into the models built in this research, and none of the existing approaches with exact procedures can be applied. Therefore, this study develops a multi-period fuzzy random simulation-