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Interfaces with Other Disciplines

Semi-analytical solutions for dynamic portfolio choice in jump-diffusion models and the optimal bond-stock mix☆

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ABSTRACT

This paper studies the optimal portfolio selection problem in jump-diffusion models where an investor has a HARA utility function, and there are potentially a large number of assets and state variables. More specifically, we incorporate jumps into both stock returns and state variables, and then derive semi-analytical solutions for the optimal portfolio policy up to solving a set of ordinary differential equations to greatly facilitate economic insights and empirical applications of jump-diffusion models. To examine the effect of jump risk on investors' behavior, we apply our results to the bond-stock mix problem and particularly revisit the bond/stock ratio puzzle in jump-diffusion models. Our results cast new light on this puzzle that unlike pure-diffusion models, it cannot be rationalized by the hedging demand assumption due to the presence of jumps in stock returns.

1. Introduction

As prompted by the seminal work of Merton (1969), there is a large literature on the dynamic portfolio choice problem that has typically been studied in continuous-time models primarily due to their analytical tractability. There are two popular methods that are widely employed to solve this problem. The first one is the HJB-based approach proposed by Merton (1969), and the other is the martingale approach advanced by Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989). In both approaches, the investor's utility function plays a fundamental role in seeking the optimal portfolio policy.[1] Unfortunately, it is well known that semi-analytical solutions to the dynamic portfolio choice problem are generally unavailable, although they are vitally important to facilitate economic insights and empirical applications. In this paper, we solve the optimal asset allocation problem in closed form for multi-asset jump-diffusion models in the way that the solutions provide a new instrument to analyze the behavior of investors with general HARA preferences towards distinct risk factors.

In a growing literature, numerous efforts have been made to solve the portfolio choice problem in closed form. Specifically, Bajeux-Besnainou and Portait (1998) extend the static setup in Markowitz (1952) to a much more challenging dynamic version and explicitly solve the dynamic mean-variance problem in a complete pure-diffusion model. Recently, by using the martingale approach, Lioui and Poncet (2016) provide closed-form solutions to the dynamic mean-variance problem in a complete affine diffusion model. As remarked by the authors, the dynamic mean-variance model in Section 2.3 of Lioui and Poncet (2016) may result in time-inconsistent portfolio strategies, showing that the investor may find it optimal to deviate from her initial policy. In contrast, Basak and Chabakauri (2010)[2] explicitly solve the time-consistent dynamic mean-variance policy based on a recursive representation. In a continuous-time mean variance model with constraints on portfolio policy, Wang and Forsyth (2011) develop a numerical scheme to determine the optimal time-consistent asset allocation strategy.[3] For a von Neumann–Morgenstern utility, Detemple, Garcia, and Rindisbacher (2003) also use the martingale approach to solve the portfolio choice problem in a complete pure-diffusion model which may include a large number of assets and state variables with non-affine structures. They obtain the optimal portfolio strategy using the Monte Carlo simulation, yet which may be

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[1] The widely used utility functions belong to the so-called hyperbolic absolute risk aversion (HARA) family, including quadratic (with restrictions on parameters), exponential, logarithmic, and power forms.

[2] We thank an anonymous referee for pointing this out to us.

[3] For a good discussion on time-inconsistent portfolio strategies, see Dang and Forsyth (2016).

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time-consuming in the presence of a large number of assets and state variables.

As discussed in Bardhan and Chao (1996), a jump-diffusion model with random jump sizes is inherently incomplete. One of the key assumptions in the aforementioned papers is the completeness of the market. In general, it is a daunting task to explicitly solve the optimal portfolio choice problem in an incomplete market. One usually resorts to either the HJB equation or the martingale method. As is well known, it is difficult to apply the HJB equation to a high-dimensional problem in both complete and incomplete markets. Furthermore, it is very challenging to use the martingale method in an incomplete market since there are infinitely many martingale measures. To solve the optimal portfolio problem in incomplete pure-diffusion models, approximation methods are proposed in Bick, Kraft, and Munk (2013) and Haugh, Kogan, and Wang (2006), respectively. Yet, their solutions are numerically approximated and thus may suffer inaccuracy.

In contrast, by assuming quadratic conditions in pure-diffusion models, Liu (2007) explicitly solves the optimal dynamic portfolio choice problem in both complete and incomplete markets, up to the solutions to a set of ordinary differential equations (ODEs). Specifically, he solves a set of ODEs by guessing the exponential linear form of the indirect value function without simulation. This method is widely used in the asset allocation literature of pure-diffusion models nowadays. However, much less is known about the conditions that can lead to the ODE-based analytic solution to the optimal portfolio choice problem in jump-diffusion models especially when both stock prices and state variables are allowed to jump.[4] The objective of the present paper is then to generalize the aforementioned ODE-based approach in pure-diffusion models to jump-diffusion models which nest the former (e.g., Liu (2007)) as special cases.

More specifically, we first consider constant relative risk aversion (CRRA) utility functions and provide the conditions under which the indirect value function in jump-diffusion models has an exponential linear form. The indirect value function and the optimal portfolio strategy can then be obtained by solving a set of ODEs. By providing an efficient two-step approach, we further extend our ODE-based method to more general HARA utility functions given their popularity in financial economics.[5] Our results show that the indirect utility function for a HARA utility takes a form significantly different from the exponential linear one for a CRRA utility. To the best of our knowledge,[6] we are not aware of any semi-analytical solution to the dynamic asset allocation problem in jump-diffusion models where risk-averse investors face jumps in multiple risky assets and state variables. More importantly, the semi-analytical solutions may greatly facilitate economic insights and enhance our understanding of investors' behavior towards jump risks.

Our paper is closely related to the work of Jin and Zhang (2012) in that they use a decomposition approach based on an HJB equation to solve a portfolio selection problem that includes a large number of risky assets and state variables. But their state variables are pure-diffusion processes and the indirect value function is evaluated by the Monte Carlo simulation. Our paper also relates to the work of Das and Uppal (2004) and Aït-Sahalia, Cacho-Diaz, and Hurd (2009). These studies solve the portfolio selection problem in jump-diffusion models, but without state variables. In contrast, we obtain semi-analytical solutions to the optimal portfolio strategy under jump-diffusion models that include a large number of assets and state variables. These solutions therefore allow us to solve in a computationally efficient way the dynamic portfolio selection problem in jump-diffusion models where both stock returns and state variables can jump.

By using the theoretical framework developed in this paper, we study the problem of how jumps in stock returns affect the optimal cash-bond-stock portfolio in a dynamic asset allocation model where an investor can trade one stock, two bonds, and cash. Especially, we revisit the asset allocation puzzle raised in Canner, Mankiw, and Weil (1997). They document the empirical evidence that strategic asset allocation advices tend to recommend a higher bond/stock ratio for a more risk-averse investor. Several studies have attempted to explain the rationality of this puzzle. For instance, Brennan and Xia (2000) and Bajeux-Besnainou, Jordan, and Portait (2001) relate the puzzle to a hedging component in the stochastic interest rate and provide elegant solutions to the asset allocation puzzle. All of these studies assume that both the short-term interest rate and stock returns follow pure diffusion processes. Our framework generalizes these studies by incorporating jumps into stock returns and examining the role of risk aversion in determining the optimal cash-bond-stock portfolio. In particular, we show both theoretically and numerically that unlike the pure-diffusion models in Brennan and Xia (2000), Bajeux-Besnainou et al. (2001) and Lioui (2007), there is no clear-cut answer to the bond/stock ratio puzzle in jump-diffusion models even despite the aforementioned hedging assumption. In other words, the puzzle itself cannot be rationalized by the hedging assumption in the presence of jumps in stock returns. The underlying reason for this is that an investor responds distinctly to diffusion risk premium and jump risk premium when there is an increase in the investors's relative risk aversion coefficient.

In summary, our paper makes three contributions to the literature on portfolio choice. First, our work generalizes the popular ODE-based approach used in pure-diffusion models to jump-diffusion models for CRRA utility functions, which may greatly alleviate computational efforts in seeking the optimal portfolio strategy. Second, we provide an efficient two-step method for solving HARA preference-based ODEs. This then extends the applicability of our approach within a family of general utility functions. Finally, we illustrate that the hedging assumption in pure-diffusion models fails to resolve the asset allocation puzzle in jump-diffusion models, which further provides a new channel for us to understand the nature of this well-known puzzle.

The rest of the paper is organized as follows. In Section 2, we present the framework for Merton's dynamic portfolio selection problem in jump-diffusion models and then present affine conditions in the jump-diffusion models. In Section 3, we use the affine conditions to explicitly solve the indirect value function and the optimal portfolio strategy in terms of the solutions to a set of ODEs for general HARA preferences. In Section 4, we

[4] Mounting empirical evidence suggests that the jump risk needs to be captured in asset price processes and other risk factors, such as volatility processes, in addition to the diffusion risk. For example, Eraker, Johannes, and Polson (2003) and Eraker (2004) among many others find strong evidence for co-jumps in volatility and stock returns, i.e., that a big jump in stock prices is likely to be associated with a big jump in volatility. Besides, Das (2002) shows that a class of Poisson–Gaussian models offer a good statistical description of short rate behavior and capture empirical features of the data which would not be captured by Gaussian models (We thank an anonymous referee for bringing this issue to our attention). In the meantime, it is well understood that jump risk in stock prices has a substantial impact on portfolio selection, see, for example, Liu, Longstaff, and Pan (2003) and Das and Uppal (2004).

[5] More importantly, Perets and Yashiv (2016) show that the HARA utility is more fundamental to economic analysis. This functional form is the unique one which satisfies basic economic principles in an optimization context. Therefore, the use of HARA utility functions is not just a matter of convenience or tractability, but rather emerges from economic reasoning, i.e., it is inherent in the economic optimization problem.

[6] It should be noted that for the logarithmic utility maximization under jump diffusions, semi-analytical solutions are generally available primarily due to its myopic nature of the optimal portfolio strategy. For example, in a general semimartingale market model, Goll and Kallsen (0000) explicitly solve the problem of maximizing the expected logarithmic utility from consumption or terminal wealth. We thank an anonymous referee for suggesting this discussion.