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Optimal portfolio choice with loss aversion over consumption

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ABSTRACT

This paper analyses the consumption–investment problem of a loss averse investor with an s-shaped utility over consumption relative to a time-varying reference level. Optimal consumption exceeds the reference level in good times and descends to the subsistence level in bad times. Accordingly, the optimal portfolio is dominated by a mean–variance component in good times and rebalanced more aggressively toward stocks in bad times. This consumption–investment strategy contrasts with customary portfolio theory and is consistent with several recent stylized facts about investor' behavior. I also analyze the joint effect of loss aversion and persistence of the reference level on optimal choices. Finally, the strategy of the loss-averse investor outperforms the conventional Merton-style strategies in bad times, but tends to be dominated by the conventional strategies in good times.

1. Introduction

In their seminal articles Kahneman and Tversky (1979, 1991, 1992) find that in many situation agents' behavior diverges from the standard parading of classical portfolio choice (Merton, 1969). More precisely, agents seem to be more sensitive to losses than gains (loss aversion), risk-averse over gains and risk seeking over losses (risk seeking behavior), rather than purely risk-averse as postulated by the classical portfolio theory. Loss aversion and risk seeking behavior, which are analyzed in this paper, are mathematically represented by the asymmetric s-shaped utility function, which is convex in the domain of losses and concave in the domain of gains.

In reaction to the findings of Kahneman and Tversky (1979, 1991, 1992) the theory of portfolio choice has been progressively extended to account for loss aversion and risk seeking behavior. The general continuous-time portfolio problem of an investor who maximizes the cumulative prospect theory utility function[1] of final wealth is studied by Jin and Zhou (2008) and He and Zhou (2011). Reichlin (2013) analyzes the problem of maximizing the non-concave utility of final wealth and provides the conditions under which the optimization problem can be solved by the concavification method. The portfolio problem of an investor with s-shaped utility over final wealth is analyzed by Berkelaar, Kouwenberg, and Post (2004). Hens and Vlcek (2011) study the relation between loss aversion and the disposition effect. Fortina and Hlouskovaa (2011) examine the investment strategy of linear loss-averse investors for different dependence structures of assets returns, namely, Gaussian copula and Clayton copula. Frühwirth and Mikula (2008) study the saving plans of loss-averse investors. Gomes (2005) solves the investment problem of a loss-averse investor in a model with two states of the world and studies the implications of loss aversion for the trading volume.

The papers described in the paragraph above consider the problem of loss aversion over final wealth or loss aversion over stock returns. However, Kahneman and Tversky (1979) argue that loss aversion is a general mental framework that can be applied to any kind of choice whose outcomes can be coded in terms of gains and losses, and not only to lotteries that involve monetary outcomes (the typical example is the return of risky assets) as is customary in the literature on loss aversion. For this reason a strand of research applies the concept of loss aversion to consumption choice. Bowman, Minehart, and Rabin (1999) study the optimal consumption–saving problem in a two period model where the agent is loss-averse over intertemporal consumption. Yogo (2008) uses a general equilibrium model with loss aversion over consumption to solve the equity premium and the risk free rate puzzle. Andries (2015) considers a general equilibrium model where the representative agent is equipped with Epstein–Zin utility and is loss-averse over the expected value of future consumption. The model generates a negative premium for skewness and a security market line flatter than the CAPM. De Giorgi and Legg (2012) analyze portfolio choice and asset prices when agents feature narrow

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[1] Cumulative prospect theory includes loss aversion, risk seeking behavior and probability distortion.

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framing and are equipped with cumulative prospect theory utility defined over gains and losses in stock returns. Curatola (2015) shows that loss aversion over inter-temporal consumption is able to reproduce, simultaneously, the observed upward sloping term structure of interest rates and the downward sloping term structure of equity. The previous papers do not analyze the optimal investment in risky assets when agents are loss averse over consumption. There are however some exceptions. Fortin, Hlouskova, and Tsigaris (2016) consider a 2-period 2-state model where agents are loss averse over consumption relative to an exogenous reference point and analyze the implication of loss aversion for portfolio choice. van Bilsen, Laeven, and Nijman (2014) solve the optimal consumption–portfolio problem of an agent equipped with s-shaped utility over consumption when the reference level is endogenous and depends on the entire stream of past consumption, as is customary in the habit formation literature.

The habit formation literature typically assumes that the reference level of consumption increases when current consumption exceeds the reference level and decreases otherwise. Instead, I assume that the reference level increases when current consumption exceeds the initial consumption and decreases otherwise. This assumption creates an endogenous link between past consumption gain/losses and current portfolio choice which is explored in this paper. The optimal consumption of the loss-averse investor resembles the pay-off a portfolio that comprises a zero-coupon bond and a binary option written on the state-price density. The zero coupon bond ensures that consumption does not descend below the minimum subsistence level. When the state-price density is below a given threshold (i.e., in good times), the binary option yields a positive pay-off that enables the investor to consume above his/her time-varying reference level. When the state-price density is above the previous threshold (i.e., in bad times) consumption descends to a constant subsistence level. As a result, the optimal wealth of the loss-averse investor can be expressed as a portfolio of zero-coupon bonds and binary options with different maturities. The binary nature of consumption implies that the investor's optimal portfolio has two components. The first component is the standard mean–variance portfolio. The second component is a gambling portfolio consisting of aggressive investments in stocks. The weight assigned to the two components changes over time depending on the investor's consumption needs. In good times, the loss-averse investor desires to smooth consumption, and thus, the optimal portfolio is dominated by the mean–variance component that guarantees that the fraction of wealth invested in bonds is sufficient to ensure that consumption remains above the reference level. In bad times, the consumption-smoothing strategy is abandoned, and the optimal portfolio is rebalanced toward stocks in order to maximize the probability that future consumption exceeds the reference level. These results fit well with some recent findings on investors' behavior. For instance, Hoffmann and Scherbakova-Stewen (2011) find that the consumption smoothing of U.S. households increases in good times and decreases in bad times. Recently, Dorn and Weber (2013) showed that many individual investors reacted to the financial crisis by increasing their equity positions. These findings are difficult to rationalize in a model with risk-averse agents but are consistent with the optimal consumption/portfolio strategy of investors equipped with an s-shaped utility of consumption.

Historical dependence in the reference level of consumption has an important and non-trivial effect on the investor's portfolio policy. Past consumption gains/losses are linked to the current reference level and, thus, determine the investor's willingness to take on risk. When past consumption is high compared to the reference level, an increase in the importance of past consumption reduces risk aversion in good times and decreases risk taking incentives in bad times. As a result, the loss-averse investor increases the fraction of wealth invested in stocks (bonds) in good (bad) times. Loss aversion also affects portfolio performances. In good times, the loss-averse investor implements a conservative investment strategy in order to ensure that consumption stays above the reference level. This strategy requires higher investments in bonds than, for instance, the investor with CRRA utility and depresses portfolio performance because bonds pay, on average, less than stocks. However, in bad times, the loss-averse investor reduces consumption to the minimum subsistence level and invests more aggressively in stocks. As a result, the portfolio's performance increases compared to the CRRA investor.

The rest of the paper is organized as follow: Section 2 presents the framework of the optimization problem, the financial market and the investor' utility function. Section 3 shows the optimal consumption–portfolio rules. Section 4 discuss the implications of loss aversion and habit formation for consumption and portfolios. Section 5 concludes.

2. The investor's problem

The economy The economy has a finite time horizon T. The uncertainty is represented by a filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P}\right)$ on which I define a d-dimensional Brownian motion $B(t) = [B_1(t), B_2(t), \ldots, B_d(t)]'$. Consider a continuous-time financial market endowed with d risky assets (stocks), indexed by $i = 1, 2, \ldots, d$, and a risk-free asset (bond). The investor can trade without transaction costs. The price of the risk free asset is denoted by $S_0(t)$ and evolves as

$$\frac{dS_0(t)}{S_0(t)} = r(t)dt \tag{1}$$

where $r(t)$ is the instantaneous risk-less interest rate. The price of the stock i follows an Ito process of the form

$$\frac{dS_i(t)}{S_i(t)} = \mu_i(t)dt + \sum_{j=1}^{d} \sigma_{ij}(t)dB_j(t) \tag{2}$$

where μ_i is the expected return and $\{\sigma_{ij}\}_{j=1,\ldots,d}$ represents the set of volatility coefficients of stock i. Assume that r, $\mu = [\mu_1, \ldots, \mu_d]$ and $\sigma = \{\sigma_{ij}\}_{1\le i,j\le d}$ are bounded and $\mathcal{F}$-adapted and that σ is an invertible and bounded matrix. Under these assumptions, the price of risk process

$$\theta = \sigma(t)^{-1}[\mu(t) - r(t)] \tag{3}$$

exists and is bounded. As a result, the state-price density of the economy is given by

$$H(t) = \exp\left(-\int_0^t r(s)ds - \frac{1}{2}\int_0^t \|\theta(s)\|^2 ds - \int_0^t \theta'(s)dB(s)\right) \tag{4}$$

with $H(0) = H_0 = 1$.

Preferences The loss-averse investor is equipped with the following s-shaped utility of consumption

$$U(c(t), Z(t)) = \begin{cases} -B\dfrac{(Z(t) - c(t)))^{1-\gamma}}{1-\gamma}, & \text{if } c(t) < Z(t), \\[2ex] \dfrac{(c(t) - Z(t))^{1-\gamma}}{1-\gamma}, & \text{if } c(t) \ge Z(t), \end{cases} \tag{5}$$