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Markets with heterogeneous beliefs: A necessary and sufficient condition for a trader to vanish

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ABSTRACT

What does it take to survive in the market? Previous literature has proposed sufficient conditions for a trader to vanish, which depend on pairwise comparisons of traders' discounted beliefs. We propose a novel condition that focuses on the ratio of traders' discounted beliefs and (approximate) equilibrium prices. Unlike existing conditions, ours is both necessary and sufficient for a trader to vanish and delivers the exact rate at which vanishing traders lose their consumption shares. As an application, we analyze the performance of two intuitive behavioral strategies: the *"Follow the Leader Strategy"* that prescribes mimicking the beliefs of the most successful trader, and the *"Follow the Market Strategy"* that prescribes to use beliefs which coincide with the state price density. Further, we show that the relative performance of vanishing traders cannot be studied in isolation. Our analysis highlights an intuitive point obscured by the existing conditions: trading in financial markets is qualitatively different from bilateral trading.

1. Introduction

More than a half-century ago economists hypothesized that traders with poor forecasting abilities progressively lose wealth against traders with more accurate probabilistic views (*market selection hypothesis*, Friedman, 1953). In general equilibrium settings with complete markets and bounded aggregate endowment, previous literature has formalized this hypothesis and found sufficient conditions for a trader to vanish based on the pairwise comparison of traders' discounted beliefs. A trader vanishes if there is another trader who is more accurate (Blume and Easley, 1992; 2006; Kogan et al., 2017; Massari, 2017; Sandroni, 2000; Yan, 2008).

This approach greatly simplifies the analysis of heterogeneous-belief economies because it does not require solving for the competitive equilibrium – traders' discounted beliefs are exogenous. However, it overlooks a fundamental aspect of competition: in financial markets, each trader interacts with all traders in the market simultaneously through prices, not in a pairwise fashion (Jouini and Napp, 2007). Therefore, in economies with more than two traders, pairwise comparisons cannot deliver a necessary and sufficient condition for a trader to vanish (Blume and Easley, 2009) or characterize the relative performance between vanishing traders (Cvitanic and Malamud, 2009).

To account for these shortcomings, we propose a novel approach that is closer to the actual trading experience in financial markets. We focus on the ratio between traders' discounted beliefs and equilibrium prices. Our approach preserves the central role of prices in market interactions and delivers a necessary and sufficient condition for a trader to vanish which

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also characterizes the exact rate at which vanishing traders lose their wealth. Moreover, it brings the general equilibrium analysis closer to the temporal equilibrium analysis of market selection – the latter focusing on the ratio between traders' investment strategies and equilibrium prices (Bottazzi and Dindo, 2014; Chiarella and He, 2001; Evstigneev et al., 2002).

To make our condition applicable without solving for the competitive equilibrium, we provide an approximation of equilibrium prices that only depends on exogenous quantities. Our main technical contribution is demonstrating that, under standard assumptions, asymptotic equilibrium prices are well approximated by a convex combination of traders' discounted beliefs. This result proves a longstanding conjecture in economics (e.g. Blume and Easley, 1993): equilibrium prices, a risk-adjusted average of traders' beliefs, are qualitatively Bayesian, a non-risk-adjusted average of probabilities.

If all traders have the same discount factor, our condition reads: a trader vanishes if and only if his beliefs are less accurate than the probability obtained via Bayes' rule from a regular prior on the set of traders' beliefs.[1] By contrast, existing conditions read: a trader vanishes if there is another trader who is more accurate. Because traders' beliefs are exogenous and Bayesian inference is well understood, our condition is easy to verify and compute even in economies with a large number of traders – a case in which conducting pairwise comparisons between all traders in the economy might prove computationally challenging.

This paper provides two novel implications. First, our condition makes it possible to analyze the performance of traders who hold non-standard beliefs or use non-belief-based investment strategies. Unlike existing conditions, ours precisely indicates which strategies vanish/survive when there are no perfectly rational traders in the market. As an illustration, we study the performance of two intuitive strategies: the *Follow the Leader Strategy* and the *Follow the Market Strategy*, henceforth FLS and FMS, respectively.

In a log-economy with homogeneous discount factors, the FLS coincides with mimicking the trader in the market that had the highest growth of capital from a given date. This intuitive strategy is easy to implement, and it is offered by a growing number of internet brokers.[2] The appeal of the FLS is that it guarantees to perform almost as well as the trader with the highest capital growth, the leader, provided that leaders do not change "too often". However, what happens when leaders do change "too often"? Conventional wisdom argues that the FLS would not perform well because of the transaction costs associated with changing the investment style. Here we show that the FLS leads to ruin even if there are no transaction costs. In Section 6.1, we illustrate this point in a three-trader economy (traders 1,2 and FLS-trader). In the example, traders 1 and 2 alternate infinitely often as a leader because they are equally (in)accurate. The FLS-trader vanishes by the following logic. (i) Every time a leader changes, the FLS-trader starts copying the new leader with a small delay, because the new leader must first outperform the previous leader. (ii) During these delays, the FLS-trader does worse than the new leader, because he is still following the previous leader. And (iii), the delay-induced losses cumulate over time resulting in ruin. This argument is not compatible with the standard pairwise comparison approach because it requires comparing the FLS-trader's performance against that of more than one trader (leaders change over time). Among the existing conditions for a trader to vanish, Sandroni's (2000) condition fails to recognize that the FLS-trader vanishes; Blume and Easley's (2006) condition incorrectly implies that the FLS-trader dominates;[3] and Blume and Easley's (2009) condition is inapplicable.[4]

The FMS prescribes adopting the next-period state price density as a belief. This strategy is behaviorally motivated by the *Wisdom of The Crowds* argument which states that equilibrium prices might reflect beliefs that are more accurate than the beliefs of all market participants (Galton, 1907). Our analysis of the FMS shares some similarities with the analysis of the FLS: the fate of the FMS-trader depends on the presence of a unique leader among the other traders in the economy. However, unlike the FLS-trader, there are situations in which the FMS-trader dominates the market. In homogeneous discount factor, CRRA-economies with no aggregate risk in which all traders are more (less) risk averse than log, the FMS-trader dominates (vanishes) if leaders among the other traders change infinitely often.[5] The intuition goes as follow. In CRRA economies, equilibrium prices can be decomposed into a belief component (the state price density), and an endogenously determined discount factor which depends on traders' beliefs and consumption shares dispersion. When $\gamma < (>)1$ the IES coefficient is larger (smaller) than in the log-utility case ($\gamma = 1$). As a result the demand for saving is high, interest rates are low, and the market discount factor is larger than traders' (common) discount factor: β. The FMS-trader uses the state price density to buy a constant share of the aggregate endowment, thus, his fate depends on the difference between his discount factor (β) and the market discount factor. If the consumption share/beliefs distribution quickly becomes degenerate, the endogenous component of the discount factor vanishes fast and has no effect on survival. However, if leaders alternate infinitely often, there is an infinite number of periods in which the consumption share/beliefs distribution is not degenerate. Therefore, the FMS-trader discount factor differs from the market's infinitely often and his fate depends on the sign of their difference.

Related papers that studies the effect of long run heterogeneity on equilibrium prices includes Jouini and Napp (2010), that analyze the long-run risk-return relationship in an economy with two agents have equally biased (constant) beliefs;

[1] A prior is regular if it attaches strictly positive probability to every probability in its prior support.

[2] E.g. signal-trader, collective2 and e-toro: https://www.youtube.com/watch?v=GX041XotWd4.

[3] It is known that Blume and Easley's (2006) Theorem 8 can lead to incorrect conclusions (Massari, 2013).

[4] The elegant geometric construction behind Blume and Easley's (2009) necessary and sufficient condition for a trader to vanish can only be used in economies in which the distribution of states' and traders' beliefs are iid.

[5] We assume constant aggregate endowment to avoid known biases due to the interaction between risk attitudes and fluctuations of the aggregate endowment.