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Decision Support

A linear programming approach to efficiency evaluation in nonconvex metatechnologies

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ABSTRACT

The notions of metatechnology and metafrontier arise in applications of data envelopment analysis (DEA) in which decision making units (DMUs) are not sufficiently homogeneous to be considered as operating in the same technology. In this case, DMUs are partitioned into different groups, each operating in the same technology. In contrast, the metatechnology includes all DMUs and represents all production possibilities that can in principle be achieved in different production environments. Often, the metatechnology cannot be assumed to be a convex set. In such cases, benchmarking a DMU against the common metafrontier requires implementing either an enumeration algorithm and solving a linear program at each of its steps, or solving an equivalent mixed integer linear program. In this paper we show that the same task can be accomplished by solving a single linear program. We also show that its dual can be used for the returns-to-scale characterization of efficient DMUs on the metafrontier.

1. Introduction

Homogeneity of decision making units (DMUs) is a common assumption made in standard applications of data envelopment analysis (DEA). This assumption allows the DMUs to be regarded as members of the same production technology, which in turn allows the DMUs to be benchmarked with respect to the common production frontier that such DMUs generate (Cooper, Seiford, & Tone, 2007). The homogeneity assumption usually means that all DMUs have similar access to the same type of resources (inputs) and produce the same range of products or services (outputs). It also implies that the operating environments of all DMUs are sufficiently similar for the purposes of efficiency evaluation (Dyson et al., 2001).

In many applications, the assumption of homogeneity may be problematic. O’Donnell, Rao, and Battese (2008) give several possible reasons for this, including differences in access to labor and financial capital, access to markets, natural environment, and other environmental characteristics.

For applications in which the homogeneity of DMUs cannot be accepted, Battese, Rao, and O’Donnell (2004) and, specifically for DEA, O’Donnell et al. (2008) develop the *metafrontier* approach that enables analysis of efficiency of heterogeneous DMUs. According to this methodology, all DMUs are classified into several groups. DMUs in the same group are considered sufficiently homogeneous and represent the same group technology. The latter are usually modeled as the constant (CRS) or variable (VRS) returns-to-scale technologies (Banker, Charnes, & Cooper, 1984; Charnes, Cooper, & Rhodes, 1978). The *within-group* efficiency of DMUs is measured against the corresponding group technology frontier.

The *metatechnology* includes all production possibilities achievable in different environments. In particular, it includes each of the above group technologies as a subset. The boundary of the metatechnology is referred to as the metafrontier. The latter can be viewed as representing the best production possibilities that can be achieved in principle, by assuming that the operating environment for DMUs can be changed.

The efficiency of a DMU measured against the metafrontier is referred to as its *meta-efficiency*. O’Donnell et al. (2008) suggest that the gap between the within-group efficiency and its meta-efficiency is interpretable as an indicator of the restrictive nature of the group’s operating environment. Kerstens, O’Donnell, and van de Woestyne (2015) provide an updated overview and discussion of the metafrontier methodology.

In recent years, the use of metafrontiers has become well-established in DEA. As highlighted by O’Donnell et al. (2008), there are two distinctly different ways in which the metatechnology, and the metafrontier, could be defined, and there are reported applications of DEA that follow both definitions.

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First, the metatechnology may be defined as, for example, the single VRS or CRS technology generated by all DMUs from all groups. This approach results in a convex metatechnology. For this approach, calculation of meta-efficiency is unproblematic and requires solving a standard VRS or CRS model on the data set that includes all DMUs in all groups. For example, this approach was implemented by Kontolaimou and Tsekouras (2010), Portela, Thanassoulis, Horncastle, and Maugg (2011), Zhang, Zhou, and Choi (2013) and Zhang and Wei (2015).

Second, the metatechnology may be defined as the union of all group technologies. Even though each group technology may be a convex set, the metatechnology defined as the union of such sets is generally not convex. For this approach, the meta-efficiency of each DMU can be obtained by either implementing an enumeration algorithm, each step of which requires solving a linear program, or solving an equivalent mixed integer linear program (Cooper et al., 2007; Huang, Ting, Lin, & Lin, 2013; Tiedemann, Francksen, & Latacz-Lohmann, 2011).[1]

As argued by Cooper et al. (2007, p. 231), Tiedemann et al. (2011) and Asmild (2015), assuming that the group technologies are convex (which is implied by the VRS or CRS models) does not mean that convex combinations of DMUs from different groups are meaningful. In particular, Asmild (2015) notes that the interpretation of benchmarks located on the metafrontier and constructed from DMUs from different operating environments may be problematic. Kerstens et al. (2015) argue that the large majority of reported applications use a convex metatechnology, which may result in a “potentially poor approximation of the metafrontier” and introduce bias in the evaluation of meta-efficiency.

It may therefore appear that the approach to modeling the metatechnology which does not assume convexity between groups, should, as argued for by Kerstens et al. (2015), be more widely acceptable in DEA applications. However, this approach is computationally less straightforward than the approach for which the meta-efficiency is evaluated by solving a single conventional VRS or CRS model, using standard DEA software.

In our paper, we address the above problem of practical use of nonconvex metatechnologies, by developing a *linear programming* approach for the evaluation of meta-efficiency of DMUs, without assuming convexity between groups.[2] From a practical perspective, this approach should be attractive because, for each DMU under the evaluation, we solve only one linear program which always has a finite optimal solution. In contrast, using a standard enumeration algorithm for the same purpose requires solving several linear programs, one for each group technology, and correctly processing all occurrences of unbounded optimal solutions or infeasibility notifications, which needs certain programming expertise and adds to the complexity of batch processing algorithms. In other words, the advantage of the proposed approach is in the simplicity of its practical application.

The suggested linear programming approach allows the dual formulation which has a meaningful interpretation. In DEA, the dual is often used for the returns-to-scale (RTS) characterization of DMUs in the VRS technology. We show that, for metafrontiers, the RTS characterization depends on the set of group frontiers on which the DMU under the evaluation is located. We show how the dual linear program can be used for the identification of all such group frontiers, which we further use to introduce a practical approach for the RTS characterization of efficient metafrontiers.

This paper is structured as follows. In Section 2, we review the idea of modeling the metatechnology as a nonconvex set and briefly introduce the existing enumeration and mixed integer linear programming approaches to efficiency evaluation in this context. In Section 3, we develop a new linear programming approach to efficiency evaluation in a nonconvex metatechnology. In Section 4, we obtain the equivalent dual multiplier programs and discuss their meaning. In Section 5, we develop a modification of the dual program that allows us to identify all group frontiers on which the DMU is projected. In Section 6, we make further use of the dual by developing the notion of scale elasticity and returns-to-scale characterization of DMUs on the metafrontier. In Section 7, we extend our results to evaluation approaches based on directional distance functions. In Section 8, we present a numerical example illustrating the calculation of scale elasticity and assessment of returns to scale for a metafrontier. In Section 9, we summarize our contribution and outline further research avenues.

2. Preliminaries

Let a number of observed DMUs, be involved in a production process, characterized by inputs $i = 1, \dots, m$ and outputs $r = 1, \dots, s$. Suppose that these DMUs operate under generally different conditions that have an effect on their production performance. Depending on the context, these may include natural, labor or regulatory environments, different access to resources, and other characteristics. To represent such differences, we assume that all DMUs can be partitioned into $G > 1$ distinct groups, so that DMUs in the same group $g \in \mathcal{G} = \{1, \dots, G\}$ operate in similar conditions.

Let each group $g \in \mathcal{G}$ include DMUs $(X_j^g, Y_j^g) \in \Re_+^m \times \Re_+^s$, $j = 1, \dots, \delta_g$, where $X_j^g = (x_{1j}^g, x_{2j}^g, \dots, x_{mj}^g)$ and $Y_j^g = (y_{1j}^g, y_{2j}^g, \dots, y_{sj}^g)$ are nonnegative and nonzero vectors[3] of inputs and outputs, respectively.

Following O’Donnell et al. (2008), we view each group of DMUs as operating in a different production technology T^g, $g \in \mathcal{G}$. To be specific, below we assume that all such technologies are VRS technologies. This assumption is not essential: we comment on technologies for other returns-to-scale assumptions in Remark 2 below. Using the conventional model of a VRS technology (Banker et al., 1984), we define each group technology as follows:

Definition 1. Technology T^g, $g \in \mathcal{G}$, is the set of pairs of vectors $(X, Y) \in \Re_+^m \times \Re_+^s$ for which there exists a vector $\lambda \in \Re_+^{\delta_g}$ such that the following conditions are true:

$$\sum_{j=1}^{\delta_g} \lambda_j^g x_{ij}^g \le x_i, \quad \forall i$$
$$\sum_{j=1}^{\delta_g} \lambda_j^g y_{rj}^g \ge y_r, \quad \forall r$$
$$\sum_{j=1}^{\delta_g} \lambda_j^g = 1.$$

Let DMU$_o$ denote the DMU (X_o^q, Y_o^q) which belongs to technology T^q, $q \in \mathcal{G}$, and whose efficiency is being evaluated. To be specific, below we consider the case of input radial efficiency. The case of output radial efficiency is similar and is only briefly discussed in Remark 3 below.

The *within-group* input radial efficiency of DMU$_o$ may be evaluated against the frontier of any individual technology T^g, $g \in \mathcal{G}$. It is found as the optimal value of the following linear program:

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[1] De Witte and Marques (2009) use the metafrontier approach for Free Disposal Hull (FDH) group technologies (Deprins, Simar, & Tulkens, 1984). In this case the metatechnology is also nonconvex.

[2] Our paper can be seen as continuing the tradition of linearizing different nonconvex production technologies, such as FDH. Examples of such approaches are discussed by Agrell and Tind (2001) and Leleu (2006).

[3] This allows some, but not all, components of vectors X_j^g and Y_j^g to be equal to zero.

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