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Abstract

In this paper we consider robust linear programs with uncertainty sets defined by the convex hull of a finite number of $m \times n$ matrices. Embedded within the matrices are related robust linear programs defined by the rows, columns, and coefficients of the matrices. This results in a nested set of primal (and dual) linear programs with predictably different optimal objective values. The set of matrices also embed a covariance structure for the matrix coefficients and we show that when negative covariances predominate in the rows, more favorable optimal objective values for the primal can be expected.

Background and Introduction

Underlying any linear programming problem, max $c \cdot x$ subject to $Ax \leq b$, $x \geq 0$, is the need to determine the coefficients of the objective vector c, the technological coefficient matrix A and the right hand side resource b. The applicability of this model to an astounding diversity of problem areas is remarkable, (see [14], [15], [16], [17], [21], [22], [23]). The diversity of applications, however, is contrasted with the uniform commonality that each problem, in whatever context, requires one to specify numerical values for c, A and b.

In the early years of linear programming, the 1950s and even before, solution methodologies and concerns about convergence overshadowed the issue of "where" or "how to collect" the needed data since problem sizes were small. But even in these early days the issue of data uncertainty emerged with the development of probabilistic constraints [9] and two-stage programming with recourse [10].These stochastic approaches to managing data uncertainty have matured and have given rise over the years to many successful applications, especially the two stage methodologies e.g. [12], [18], [28]. Applications of stochastic programming to finance, manufacturing, telecommunications transportation, power generation and crew assignment are highlighted in the well-known text [8]. But the stochastic models add significant burdens both in computational complexity and data acquisition. Models with probabilistic constraints typically result in non-convex programs and/or rely on probabilistic independence of critical parameters. Two-stage models, while convex, typically are very large, although innovative solution strategies with decomposition [20] greatly reduce