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Weak Interactions Based System Partitioning Using Integer Linear Programming

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Abstract: The partitioning of a system model will condition the structure of the controller as well as its design. In order to partition a system model, one has to know what states and inputs to group together to define subsystem models. For a given partitioning, the total magnitude of the interactions between subsystem models is evaluated. Therefore, the partitioning problem seeking for weak interactions can be posed as a minimization problem. Initially, the problem is formulated as a non-linear integer minimization that is then relaxed into a linear integer programming problem. It is shown within this paper that cuts can be applied to the initial search space in order to find the least interacting partitioning; only composed of controllable subsystems. Two examples are given to demonstrate the methodology.

Keywords: Decoupling problems, linear multivariable systems, decentralized control, integer programming.

1. INTRODUCTION

Systems models are widely used in control design especially with the development of techniques such as model predictive control (Rawlings and Mayne (2009)) (Maciejowski (2002)). Systems are growing in size and complexity and they are in most cases composed of interacting subsystems (Scattolini (2009)). For these large scale systems, the design of a centralized controller can be prohibitive due to the heavy computational resources required (Mayne (2014)). Also, if the system is geographically spread out, communication delays between the centralized controller and the actuators and sensors arise. One way to solve this problem is to see the system as a concatenation of subsystems and to design local controllers for each subsystem. In a top-down approach the full model of the multivariable system is partitioned into subsystem models so that the decentralized controller can be designed. Decentralized control has been studied for decades and design procedures have been established (Šiljak (1991)) (Bakule (2008)). However, the system model partitioning problem has been overlooked, often because the system is already composed of physical subsystems. Every subsystem model is defined by a set of states and inputs. The weak interaction partitioning problem consist of defining these sets in order to minimize the coupling between subsystem models. For instance, strongly coupled subsystem models can emerge from the main system model, particularly within chemical plants (Stewart et al. (2010)) or heating systems (Moroşan et al. (2010)). The ideal partitioning of a system model would yield completely decoupled subsystem models.

Defining the subsystems of a plant has been done in different ways in the past. One of the first methods employed to couple inputs and outputs was the relative gain array (Bristol (1966)). This method is used to find the best pairing at steady state between inputs and outputs and hence to choose the most relevant input to control a given output in a multi-input multi-output system. It can be seen as a response to the industrial need to control a multi-variable process as a combination of single variable processes. The relative gain array has been extended to the block relative gain, allowing for suitable pairing for block decentralized control (Manousiouthakis et al. (1986)) (Kariwala et al. (2003)). The extension of the relative gain array allows the design of multi-variable controllers in a decentralized way. However it only links inputs and outputs together and does not provide a partitioning of the plant model. A technique similar to the relative gain array, is the Nyquist array method, allowing the design of single-input single-output controllers after rendering the model diagonaly dominant (Leininger (1979)) (Chen and Seborg (2003)). System partitioning can be performed by seeking the least interacting groups. Another technique used for system decomposition and integration is the design structure matrix also known as the dependency structure matrix or interaction matrix (Browning (2001)). This technique indicates the link between the elements it represents, moreover the links are directed. Elements along a row indicate that a contribution is provided to other elements whereas elements along a column indicates a dependency from other

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parts of the system. The attribution of weights within the interaction matrix is used in order to perform clustering and achieve system decomposition. Other work on decentralized control combined the controller design along with the controller topology, these two aspects are combined in the optimization function yielding a trade-off between the need for feedback links and the loss of performance compared to the centralized controller (Schuler et al. (2014)). Finally, other works have studied the actuator partitioning problem (Jamoom et al. (1998)) (Motee and Sayyar-Rodsari (2003)). To the best of the authors' knowledge the problem addressing state space model partitioning has not been studied. Therefore this partitioning approach is a standalone work, making any comparison difficult.

In this paper, we propose an integer programming based approached to the problem of partitioning a system model into a set of non-overlapping but coupled subsystem models. The objective is to reduce the magnitude of the interactions between the subsystem models. Finally, cuts are added to rule out non-controllable partitionings in order for the algorithm to yield only controllable subsystem models.

The paper is organised as follows. Section 2 states the problem and section 3 introduces the required notations. Section 4 demonstrates how the problem can be relaxed into a linear integer programming problem. In section 5 the controllability cut principle is presented allowing to obtain controllable subsystems, section 6 explains the linear partitioning algorithm. In order to illustrate the algorithm section 7 includes some examples, finally section 8 concludes the paper.

Notation: For $(a, b) \in \mathbb{N}^2$ such that $a < b$, the set $[\![a; b]\!]$ defines the set containing the integers from a to b included. The operator $|.|$ is used to denote the magnitude of a complex number when applied to a complex number. When applied to a matrix the magnitude operator is applied to all the matrix elements then summed. The operator $\|.\|_2$ defines the Euclidean norm for complexes, vectors and matrices. For a set $\mathbb{N}$, the notation $\mathbb{N}^*$ defines $\mathbb{N}/\{0\}$. The superscript $^\intercal$ represents the transpose of a vector or matrix. A matrix $B \in \mathbb{R}^{n\times m}$ will be noted $(b_{ij})_{(i,j)\in[\![1;n]\!]\times[\![1;m]\!]}$ and $(B_{kl})_{(k,l)\in[\![1;N]\!]\times[\![1;M]\!]}$, respectively for the element and block notations, with N row blocks and M column blocks. For all $n \in \mathbb{N}^*$ and $A \in \mathbb{R}^{n\times n}$, $trace(A)$ denotes the sum of all the diagonal elements of A. For all $(n, m) \in (\mathbb{N}^*)^2$ and $A \in \mathbb{R}^{n\times m}$, $rank(A)$ denotes the dimension of the vector space spanned by the columns of A. For any $(i, j, k) \in (\mathbb{N}^*)^3$ and $(A, B) \in \mathbb{R}^{i\times j} \times \mathbb{R}^{i\times k}$ the notation $[A|B]$ defines the matrix obtained by concatenating A and B horizontally. For any couple of integers $(i, j) \in \mathbb{N}^2$, δ_{ij} denotes the Kronecker delta function.

2. PROBLEM STATEMENT

Given a linear time invariant controllable state space model defined by

$$\dot{x} = Ax + Bu \tag{1}$$

where the matrix A is the state matrix and the matrix B is the input matrix respectively with the appropriate sizes for N states and M inputs, therefore, $x \in \mathbb{R}^N$ and $u \in \mathbb{R}^M$. Partitioning the system model (1) consists of decomposing the inputs as well as the states into groups representing subsystems. For a given number of partitions $P \in [\![2; \min(N, M)]\!]$ and for any subsystem $p \in [\![1; P]\!]$ the model p can be expressed as follows

$$\dot{x}_p = A_{pp}x_p + B_{pp}u_p + \sum_{\substack{j=1 \\ j\neq p}}^{P} \{A_{pj}x_j + B_{pj}u_j\} \tag{2}$$

with for all $p \in [\![1; P]\!]$, $x_p \in \mathbb{R}^{N_p}$ and $u_p \in \mathbb{R}^{M_p}$ such that

$$\sum_{p=1}^{P} N_p = N \tag{3a}$$

$$\sum_{p=1}^{P} M_p = M \tag{3b}$$

The weak interaction partitioning problem consists of minimizing the magnitude of the right-hand side sum in (2) for the subsystems while keeping each of them controllable. A non-overlapping condition for the states and the inputs is imposed by (3). The next section presents the decision variables, the constraints as well as the interaction metric necessary to formulate the weak interactions optimization problem.

3. WEAK INTERACTIONS PROBLEM FORMULATION

3.1 Decision variables

A decision variable is associated to the couples formed by a group p and a state i as well as a group p and an input k. All the decision variables are binary variables. They are organised in two grouping matrices, the state grouping matrix $\alpha \in [\![0; 1]\!]^{P\times N}$ and the input grouping matrix $\beta \in [\![0; 1]\!]^{P\times M}$. Therefore, the rows of α and β represent the P groups and the columns respectively represent the N states and the M inputs. For example with $P = 3$ and $N = 5$ a non-overlapping state grouping matrix could be

$$\alpha = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In this example, the first three states belong to the second group, the fourth state composes the first group and the last state is in the third group. Hence, a specific partitioning is represented by a pair of state and input non-overlapping grouping matrices. The columns of the grouping matrices are composed of zeros and a single one. The one is positioned in the row representing the group where the state or input belongs respectively for a state and an input non-overlapping grouping matrix. The next subsection presents the linear constraints restricting the decision variables α and β in the integer optimization problem.