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Randomized Constraints Consensus for Distributed Robust Linear Programming ⋆

Mohammadreza Chamanbaz [*,***] **Giuseppe Notarstefano** [**] **Roland Bouffanais** [***]

[*] *Arak University of Technology, Arak, Iran*
`Chamanbaz@arakut.ac.ir`
[**] *Department of Engineering, Università del Salento, Lecce, Italy*
`giuseppe.notarstefano@unisalento.it`
[***] *Singapore University of Technology and Design, Singapore*
`{Chamanbaz,bouffanais}@sutd.edu.sg`

Abstract: In this paper we consider a network of processors aiming at cooperatively solving linear programming problems subject to uncertainty. Each node only knows a common cost function and its local uncertain constraint set. We propose a randomized, distributed algorithm working under time-varying, asynchronous and directed communication topology. The algorithm is based on a local computation and communication paradigm. At each communication round, nodes perform two updates: (i) a verification in which they check—in a randomized setup—the robust feasibility (and hence optimality) of the candidate optimal point, and (ii) an optimization step in which they exchange their candidate bases (minimal sets of active constraints) with neighbors and locally solve an optimization problem whose constraint set includes: a sampled constraint violating the candidate optimal point (if it exists), agent's current basis and the collection of neighbor's basis. As main result, we show that if a processor successfully performs the verification step for a sufficient number of communication rounds, it can stop the algorithm since a consensus has been reached. The common solution is—with high confidence—feasible (and hence optimal) for the entire set of uncertainty except a subset having arbitrary small probability measure. We show the effectiveness of the proposed distributed algorithm on a multi-core platform in which the nodes communicate asynchronously.

Keywords: Distributed Optimization, Randomized Algorithms, Robust Linear Programming, Optimization and control of large-scale network systems, Large scale optimization problems.

1. INTRODUCTION

Robust optimization plays an important role in several areas such as estimation and control and has been widely investigated. Its rich literature dates back to the 1950s, see Ben-Tal and Nemirovski (2009) and references therein. Very recently, there has been a renewed interest in this topic in a parallel and/or distributed framework. In Lee and Nedić (2013), a synchronous distributed random projection algorithm with almost sure convergence is proposed for the case where each node has independent cost function and (uncertain) constraint. Since the distributed algorithm relies on extracting random samples from an uncertain constraint set, several assumptions on random set, network structure and agent weights are made to prove almost sure convergence. The synchronization of update rule relies on a central clock to coordinate the step size selection. To circumvent this limitation the same authors in Lee and Nedić (2016) present an asynchronous random projection algorithm in which a gossip-based protocol is used to desynchronize the step size selection. The proposed algorithms in (Lee and Nedić, 2013, 2016), require computing projection onto the constraint set at each iteration which is computationally demanding if the constraint set does not have a simple structure such as half space or polyhedron. In Carlone et al. (2014), a parallel/distributed scheme is considered for solving an uncertain optimization problem by means of the scenario approach (Calafiore and Campi, 2004). The scheme consists of extracting a number of samples from the uncertain set and assigning them to nodes in a network. Each node is assigned a portion of the extracted samples. Then, a variant of the constraints consensus algorithm introduced in Notarstefano and Bullo (2011) is used to solve the deterministic optimization problem. A similar parallel framework for solving convex optimization problems with one uncertain constraint via the scenario approach is considered in You and Tempo (2016). In this setup, the sampled optimization problem is solved in a distributed way by using a primal-dual subgradient (resp. random projection) algorithm over an undirected (resp. directed) graph. We remark that in Carlone et al. (2014); You and Tempo (2016), constraints and cost function of all agents are identical. In Bürger et al. (2014), a cutting plane consensus algorithm is introduced for solving convex optimization problem where constraints are dis-

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tributed to the network processors and all processors have common cost function. In the case where constraints are uncertain, a worst-case approach based on a pessimizing oracle is used. The oracle relies on the assumption that constraints are concave with respect to uncertainty vector and the uncertainty set is convex. A distributed scheme based on the scenario approach is introduced in Margellos et al. (2016) in which random samples are extracted by each node from its local uncertain constraint set and a distributed proximal minimization algorithm is designed to solve the sampled optimization problem. The number of samples required to guarantee robustness can be large if the probabilistic levels defining robustness of the solution—accuracy and confidence levels—are stringent, possibly leading to a computationally demanding sampled optimization problem at each node.

The main contribution of this paper is the design of a fully distributed algorithm to solve an uncertain linear program in a network with directed and asynchronous communication. The problem under investigation is a linear program in which the constraint set is the intersection of local uncertain constraints, each one known only by a single node. Starting from a deterministic constraint exchange idea introduced in Notarstefano and Bullo (2011), the algorithm proposed in this paper introduces a randomized, sequential approach in which each node: (i) locally performs a probabilistic verification step (based on a local sampling of its uncertain constraint set), and (ii) solves a local, deterministic optimization problem with a limited number of constraints. If suitable termination conditions are satisfied, we are able to prove that the nodes agree on a common solution which is probabilistically feasible and optimal with high confidence. As compared to the literature above, the proposed algorithm has three main advantages. First, no assumptions are needed on the probabilistic nature of the local constraint sets. Second, each node can sample locally its own uncertain set. Thus, no central unit is needed to extract samples and no common constraint set needs to be known by the nodes. Third and final, nodes do not need to perform the whole sampling at the beginning and subsequently solve the (deterministic) optimization problem. Online extracted samples are used only for verification, which is computationally inexpensive. The optimization is performed always on a number of constraints that remains constant at each node and depends only on the dimension of the decision variable and on the number of node neighbors.

The paper is organized as follows. In Section 2, we formulate the uncertain distributed linear program (LP). Section 3 presents our distributed sequential randomized algorithm for finding a solution—with probabilistic robustness—to the uncertain distributed LP. The probabilistic convergence properties of the distributed algorithm are investigated in Section 4. Finally, extensive numerical simulations are performed in Section 5 to prove the effectiveness of the proposed methodology.

2. PROBLEM FORMULATION

We consider a network of processors with limited computation and/or communication capabilities that aim at cooperatively solving the following uncertain linear program

$$\begin{aligned} &\min_{\theta} \; c^T\theta \\ &\text{subject to } \; A_i^T(q)\theta \le b_i(q), \; \forall q \in \mathbb{Q}, \; i \in \{1,\ldots,n\}, \end{aligned} \tag{1}$$

where $\theta \in \Theta \subset \mathbb{R}^d$ is the vector of decision variables, $q \in \mathbb{Q}$ is the uncertainty vector, $c \in \mathbb{R}^d$ defines the objective direction, $A_i(q) \in \mathbb{R}^{m_i \times d}$ and $b_i(q) \in \mathbb{R}^{m_i}$, with $m_i \ge d$, define the (uncertain) constraint set of agent $i \in \{1,\ldots,n\}$. Processor i has only knowledge of a constraint set defined by $A_i(q)$ and $b_i(q)$ and the objective direction c (which is the same for all nodes). Each node runs a local algorithm and by exchanging limited information with neighbors, all nodes converge to the same solution. We want to stress that there is no (central) node having access to all constraints. We make the following assumption regarding problem (1).

Assumption 1. (Non-degeneracy). The minimum point of any subproblem of (1) with at least d constraints is unique and there exist only d constraints intersecting at the minimum point.

We let the nodes communicate according to a time-dependent, directed communication graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ where $t \in \mathbb{N}$ is a universal time, $\mathcal{V} = \{1,\ldots,n\}$ is the set of agent identifiers and $(i,j) \in \mathcal{E}(t)$ indicates that i send information to j at time t. The time-varying set of incoming (resp. outgoing) neighbors of node i at time t, $\mathcal{N}_{\text{in}}(i,t)$ ($\mathcal{N}_{\text{out}}(i,t)$), is defined as the set of nodes from (resp. to) which agent i receives (resp. transmits) information at time t. A directed static graph is said to be *strongly connected* if there exists a directed path (of consecutive edges) between any pair of nodes in the graph. For time-varying graphs we use the notion of *uniform joint strong connectivity* formally defined next.

Assumption 2. (Uniform joint strong connectivity). There exists an integer $L \ge 1$ such that the graph $\left(\mathcal{V}, \bigcup_{\tau=t}^{t+L-1} \mathcal{E}(\tau)\right)$ is strongly connected for all $t \ge 0$.

There is no assumption on how uncertainty q enters problem (1) making it computationally difficult to solve. In fact, if the uncertainty set $\mathbb{Q}$ is an uncountable set, problem (1) is a semi-infinite optimization problem involving infinite number of constraints. In general, there are two main paradigms to solve an uncertain optimization problem of form (1). The first approach is a deterministic worst-case paradigm in which the constraints are enforced to hold for *all* possible uncertain parameters in the set $\mathbb{Q}$. This approach is computationally intractable for cases where uncertainty does not appear in a "simple" form, e.g. affine, multi-affine, convex, etc. The second approach is a probabilistic approach where uncertain parameters are considered to be random variables and the constraints are enforced to hold for the entire set of uncertainty except a subset having an arbitrary small probability measure. In this paper, we follow a probabilistic approach and present a distributed tractable randomized setup for finding a solution—with desired probabilistic properties—for the optimization problem (1).

Notation

The constraint set of agent i is defined by

$$H^i(q) \doteq [A_i(q), b_i(q)].$$