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Improvements on Interpolation Techniques based on Linear Programming for Constrained Control ⋆

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Abstract: We present some improvements on interpolation based control (IC) for linear discrete-time systems with polyhedral constraints on the control and the states variables. The plant may be uncertain, time-varying, and subject to bounded disturbances. Roughly speaking, IC approach is based on the interpolation between an inner and an outer controller. The extension presented here is twofold. On the one hand, the IC approach is extended to deal with higher order inner controllers, making it e.g. possible to incorporate integral action in a vicinity of the origin. On the other hand, this paper presents a modification that better exploits the control signal range. As with previous IC strategies, the presented method demands relatively low online computational resources, and is applicable also to uncertain plants. The benefits of the improved scheme are illustrated in some examples.

Keywords: Constrained control; Model predictive and optimization-based control; Robust control; Time-varying systems; Linear programing

1. INTRODUCTION

This paper deals with the problem of regulating linear discrete-time systems in the presence of state and control constraints. Approaches handling problems of this type include Vertex control (Gutman and Cwikel, 1986; Blanchini and Miani, 2008), Model Predictive Control (MPC) (Mayne et al., 2000) and Interpolating Control (IC) (Nguyen et al., 2013; Nguyen, 2014).

MPC has received wide attention from the academy in the last three decades and has grown to dominate the process industry (Borrelli et al., 2015). MPC provides a systematic approach for multivariable systems with state and control constraints. The main drawback of MPC is the computational complexity involved in the optimization problem solved in each computation step. This limitation is dealt with by tailored optimization tools (Wang and Boyd, 2010), by pre-computing the optimization results for each state and storing them in a look-up table (Bemporad et al., 2002), or by interpolation based methods, unrelated to IC, (Rossiter and Ding, 2010). However, extensions of these algorithms to deal with plant uncertainty involve excessive computations, thus holding up implementations.

Vertex control can be implemented to the same type of plants, and requires low computational demands. The main disadvantage of Vertex control is that it only uses the full control range at the boundary of a predefined controlled invariant set. This leads to slow convergence when compared with MPC. To overcome this disadvantage one may switch to a more aggressive local controller near the origin, but this leads to a non-smooth control signal.

The control scheme in Nguyen et al. (2013) overcomes this shortcoming by interpolating between an "inner" linear controller designed for performance, around the origin, and an "outer" stabilizing control law (e.g., Vertex control and Minkowski functional minimization control (Blanchini and Miani, 2008)) that enlarges the admissible set. This IC method requires the solution of at most two Linear Programming (LP) problems at each time step, opposed to the Quadratic Programming (QP) in MPC, making it computationally attractive. IC is easily extendable to control uncertain and time-varying plants with negligible increase in the online computational burden (Nguyen et al., 2011).

This work further improves the IC, and relies on the recent work documented in Gutman and Nguyen (2014). Two improvements are presented: (i) handling the interpolation

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of inner and outer controller with different number of states, and (ii) better exploiting the control range.

The first improvement allows for a controller of any order to be used in proximity to the origin with negligible increase in computation time. MPC on the other hand, always produces an affine control law for any given state belonging to its region of feasibility. For example, in MPC, introducing integral action on some of the states results in having to solve a higher order QP, significantly increasing the computational load. In the proposed integrated scheme, the higher order inner controller has no effect on the order of the LP solved in each step. The second improvement guarantees the feasibility of a full control range in the controller's region of feasibility.

This work is organized as follows. Preliminaries, including the problem statement and some definitions from invariant set theory, are given in Section 2. The IC approach is briefly presented in Section 3. The main results, improvements for using a higher order inner controller and for better utilization of control signal range, are presented in Section 4. Examples are given in Section 5, and finally, the conclusions are presented in Section 6.

2. PRELIMINARIES

We consider the uncertain and/or time-varying linear discrete-time plant:

$$x(k+1) = A(k)x(k) + B(k)u(k) + Dw(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, and $w(k) \in \mathbb{R}^d$ are the measurable state, input, and disturbance vectors, respectively. The matrices $A(k) \in \mathbb{R}^{n\times n}$, $B(k) \in \mathbb{R}^{n\times m}$, and $D \in \mathbb{R}^{n\times d}$. The matrices $A(k)$ and $B(k)$ satisfy:

$$\begin{aligned} &A(k) = \sum_{i=1}^{s} \alpha_i(k)A_i,\ B(k) = \sum_{i=1}^{s} \alpha_i(k)B_i, \\ &\sum_{i=1}^{s} \alpha_i(k) = 1,\ \alpha_i(k) \geq 0,\ \forall i = 1,\ldots,s, \end{aligned} \tag{2}$$

where $\alpha_i(k)$ may be constant but unknown, random at each time, representing a linear parameter varying (LPV) plant, or time varying in some other way. See Nguyen (2014) for a more general uncertainty description that can be modeled as (2).

We consider that the state vector $x(k)$, the control vector $u(k)$, and disturbance vector $w(k)$ are subject to bounded polyhedral constraints $\mathcal{X}$, $\mathcal{U}$, and $\mathcal{W}$, respectively

$$\begin{aligned} \mathcal{X} &= \{x \in \mathbb{R}^n : F_x x \preceq g_x\}, \\ \mathcal{U} &= \{u \in \mathbb{R}^m : F_u u \preceq g_u\}, \\ \mathcal{W} &= \{w \in \mathbb{R}^d : F_w w \preceq g_w\}. \end{aligned} \tag{3}$$

The symbol $\preceq$ is used to denote element-wise inequalities.

We now give some relevant basic definitions from invariant set theory. The interested reader is referred to Blanchini and Miani (2008) for more details. For this purpose, it is assumed that there exists a linear state feedback control law

$$u(k) = Kx(k). \tag{4}$$

that robustly stabilizes system (1). The closed-loop system formed by (1) and (4) is given by

$$x(k+1) = A(k)x(k) + B(k)Kx(k) + Dw(k) \tag{5}$$

Note that this closed-loop system can represent a feedback loop with a dynamic controller, using an augmented state composed from the states of the controlled plant and the controller states (see e.g. Tarbouriech et al. (2011)).

Definition 1. Given a closed-loop system (5) with constraints (3), the set $\Omega_0 \subseteq \mathcal{X}$ is a *robustly positively invariant constrained-admissible set* if for each $x(k) \in \Omega_0$ and $w(k) \in \mathcal{W}$, it holds that $A(k)x(k) + B(k)Kx(k) + Dw(k) \in \Omega_0$ and $Kx(k) \in \mathcal{U}$.

The largest robustly positively invariant constrained-admissible set is generally called the maximal admissible set (MAS). Methods for computing the exact or polyhedral approximation of the MAS are presented in Gilbert and Tan (1991); Blanchini and Miani (2008). Henceforth the MAS will be denoted as Ω and described by its polyhedral approximation:

$$\Omega = \{x \in \mathbb{R}^n : F_o x \preceq g_o\}. \tag{6}$$

Definition 2. Given a system (1), the set $\Psi \subseteq \mathcal{X}$ is *robustly controlled positively invariant set*, if for any $x(k) \in \Psi$ and $w(k) \in \mathcal{W}$, there exists a control $u(k) \in \mathcal{U}$ such that $x(k+1) \in \Psi$. Such a control is called *admissible control*.

Definition 3. Given a system (1), the set of states $C_N \subseteq \mathcal{X}$ is an *N-step robustly controlled set* from Ω, if all states belonging to it can be steered to the set Ω in no more than N steps satisfying $u(k) \in \mathcal{U}$ for all $w(k) \in \mathcal{W}$.

Since Ω is an invariant set, C_N is a robustly controlled positively invariant set. There exist algorithms to construct C_N leading to a polyhedral set

$$C_N = \{x \in \mathbb{R}^n : F_N x \preceq g_N\}. \tag{7}$$

3. INTERPOLATION BASED CONTROL

The method presented here is built upon some previous works, see e.g. Nguyen et al. (2013) and Nguyen (2014), that present an interpolation based scheme. The main idea is to have a linear feedback law (4) designed for desired performance and robustness, admissible in a (inner) bounded set $\Omega \subset \mathcal{X}$. Then in order to enlarge the domain of attraction, it is interpolated with an affine stabilizing control law, admissible in an outer set $C_N \subseteq \mathcal{X}$, that contains Ω. These two control laws are denoted as *inner* and *outer*, respectively.

Taking Ω as the MAS of the inner controller, and C_N its N-step robustly controlled invariant set, any state $x \in C_N$ can be decomposed as

$$x(k) = c(k)x_v(k) + (1-c(k))x_o(k) \tag{8}$$

where $x_v(k) \in C_N$, $x_o(k) \in \Omega$ and $0 \leq c(k) \leq 1$. The control signal is similarly decomposed as

$$u(k) = c(k)u_v(k) + (1-c(k))u_o(k) \tag{9}$$

with $u_o(k)$ and $u_v(k)$ computed by the inner and outer control law, respectively.

Recalling that the inner controller $u_o{=}Kx_o$ was designed for desired performance and robustness, it is then desirable to have u as close as possible to u_o for all $x \in C_N$. Moreover, it is clear that having $c=0$ for $x \in \Omega$ results in an admissible control input. This motivates the following optimization