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Dynamic stiffness control of piezoelectric ring based on finite difference and hybrid programming simulation

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ABSTRACT

During the flight of hypersonic aircraft, stiffness of some local annular region needs to be enhanced. For the stiffness control system of the ring laminated with piezoelectric sensor and actuator, an annular multi-modes smart element hybrid programming simulation model has been developed to simulate and analyze the closed-loop stiffness control of independent mode. The dynamic partial differential equations of piezoelectric ring are transformed into ordinary differential equations containing sensing and control effect matrix by finite difference method. To solve the big-matrix calculation and multiple iteration problem, hybrid programming smart element simulation system is developed with C++ and MATLAB. The cosine type sensor/actuator controlling the dynamic stiffness of independent mode with displacement/acceleration feedback is employed to be a simulation example. Simulations and analysis are conducted in different conditions of differential density, integral time step and gain ratio, and relatively satisfying result is obtained. Finally, the experimental verification is carried out. This method provides a new solution to the big-data discrete simulation and analysis of complex rotary surface smart structure system.

1. Introduction

In order to reduce the mass of the aircraft, thin shell structure is employed to form the outer surface and ring or cylindrical structures are employed to form the geometrical structure. Researches has been conducted on the vibration control of ring and cylinder shell with piezoelectric smart structures [1–5]. During the studies of smart structures, simulations and analysis need to be conducted ahead of experiments, to verify the control effect or to optimize the control system. And the conventional finite element simulation method is not able to accomplish the data extraction of closed loop control system. In this paper, the hybrid programming simulation system is developed based on the closed loop piezoelectric ring.

The finite element analysis software ANSYS is employed in the literature[6] to conduct the simulations and analysis of smart piezoelectric laminated beam structure and the displacement, stress and resonant modes of the piezoelectric layer

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are calculated. But the computing time increases with the partition number ascending. Mathematic model is developed based on the piezoelectric-controlled beam and MATLAB is employed to conduct simulation in the literature [7]. And the simulation results are closed to the experiment results. But the big matrices inputting capacity of MATLAB is limited and problems like program crash appear when dealing with models in large dispersion. In the literature [8–10], analog circuit basing on the finite differential method is employed to conduct simulations of piezoelectric beam and the piezoelectric ceramics actuator of plate structure system. When developing complex systems, errors of the electronic components can be accumulated. DSP digital circuit is employed in the literature [11] to develop smart element simulation system of piezoelectric cantilever beam, and the Code Composer Studio is employed to process data to enable the system with greater control capacity. But it takes relatively more memory when dealing with matrices computing. In the existing researches [12], numerical simulations of piezoelectric smart structure are centered upon beam and plate structures, and relatively less studies are conducted on complex rotating structures. So, to solve the smart element simulation on complex rotating structures, it is necessary to build a hybrid programming simulation system with the great matrices computing capacity of MATLAB and the high iteration efficiency of C++ to accomplish the simulations of complex rotating structure in the smart piezoelectric control.

Based on the existing researches, the specific smart element simulation system for multi-mode vibration control of piezoelectric ring shell smart structure is proposed in this paper. The dynamic partial differential equations of the ring shell system in controlled or uncontrolled situations are transformed into ordinary differential equations with discrete smart element by finite difference method and the counterpart smart element matrices model is developed. The C++/MATLAB hybrid programming simulation system is built subsequently. Dynamic stiffness control with displacement/acceleration feedback is conducted based on the simulation system, and relatively desired results are obtained.

2. Major structure model of ring shell

The ring shell with PVDF (polyvinylidene fluoride) on the internal and external surface as actuator and sensor respectively is selected to be the object of this study. Thicknesses of the actuator layer and sensor layer are h^a and h^s respectively. The radium of the ring shell is R. The principal curvature coordinate system is developed on the ring with the circumferential direction to be direction θ, the axial direction to be direction x, and the radial direction to be direction α_3. To give a clear diagram, the thickness of the ring shell is amplified in the Fig. 1.

When the ring with piezoelectric sensor and actuator is in the piezoelectric-controlled situation with no external excitation, the dynamic partial differential equations of the points in circumferential direction can be written as [13]:

$$\frac{Yh}{R^2(1-\mu^2)}\left(\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{\partial u_3}{\partial\theta}\right)-\frac{\partial N^a_{\theta\theta}}{R\partial\theta}+\frac{Yh^3}{12R^4(1-\mu^2)}\left(\frac{\partial^2 u_\theta}{\partial\theta^2}-\frac{\partial^3 u_3}{\partial\theta^3}\right)-\frac{\partial M^a_{\theta\theta}}{R^2\partial\theta}=\rho h\ddot{u}_\theta \tag{1}$$

$$\frac{Yh^3}{12R^4(1-\mu^2)}\left(\frac{\partial^3 u_\theta}{\partial\theta^3}-\frac{\partial^4 u_3}{\partial\theta^4}\right)-\frac{\partial^2 M^a_{\theta\theta}}{R^2\partial\theta^2}-\frac{Yh}{R^2(1-\mu^2)}\left(\frac{\partial u_\theta}{\partial\theta}+u_3\right)+\frac{N^a_{\theta\theta}}{R}=\rho h\ddot{u}_3 \tag{2}$$

where Y is the young modulus of the ring shell; ρ is the density of the ring shell; $N^\alpha_{\theta\theta}$ and $M^\alpha_{\theta\theta}$ are the membrane force and bending moment respectively; u_θ and u_3 are respectively the real-time displacements along circumferential and radial directions.

As shown in the Fig. 1**,** the ring is segmented into m segments along the circumferential direction. The central angle of each segment is $2\pi/m$, indicated as $\Delta\theta$. Take the node at 12o′ clock as the node 0, indicated as $i = 0$. Then number the nodes between each two adjacent segments in clockwise order, indicated as $i = 1,2\ldots m$. The displacements along θ and α_3 direction of each node are indicated as u_θ, i and u_3, i respectively. When the PVDF actuator control effect is neglected, by finite difference method, dynamic partial differential equations of node i can be transformed into ordinary differential equations related to the front and rear nodes:

$$\begin{aligned}&\frac{u_{\theta,i+1}}{(\Delta\theta)^2}\left(\frac{D}{R^4}+\frac{K^*}{R^2}\right)-\frac{2u_{\theta,i}}{(\Delta\theta)^2}\left(\frac{D}{R^4}+\frac{K^*}{R^2}\right)+\frac{u_{\theta,i-1}}{(\Delta\theta)^2}\left(\frac{D}{R^4}+\frac{K^*}{R^2}\right)-\frac{u_{3,i+2}D}{2R^2(\Delta\theta)^3}+u_{3,i+1}\left[\frac{D}{R^4(\Delta\theta)^3}+\frac{K^*}{2R^2\Delta\theta}\right]\\&\quad-u_{3,i-1}\left[\frac{D}{R^4(\Delta\theta)^3}+\frac{K^*}{2R^2\Delta\theta}\right]+\frac{u_{3,i-2}D}{2R^2(\Delta\theta)^3}=\rho h\ddot{u}_{\theta,i}\end{aligned} \tag{3}$$

$$\begin{aligned}&\frac{u_{\theta,i+2}D}{2R^4(\Delta\theta)^3}-u_{\theta,i+1}\left[\frac{D}{R^4(\Delta\theta)^3}+\frac{K^*}{2R^2\Delta\theta}\right]+u_{\theta,i-1}\left[\frac{D}{R^4(\Delta\theta)^3}+\frac{K^*}{2R^2\Delta\theta}\right]-\frac{u_{\theta,i-2}D}{2R^4(\Delta\theta)^3}-\frac{u_{3,i+2}D}{R^4(\Delta\theta)^4}+\frac{4u_{3,i+1}D}{R^4(\Delta\theta)^4}\\&\quad-u_{3,i}\left(\frac{6D}{R^4(\Delta\theta)^4}+\frac{K^*}{R^2}\right)+\frac{4u_{3,i-1}D}{R^4(\Delta\theta)^4}-\frac{u_{3,i-2}D}{R^4(\Delta\theta)^4}=\rho h\ddot{u}_{3,i}\end{aligned} \tag{4}$$