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Indifference pricing of a life insurance portfolio with risky asset driven by a shot-noise process

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ABSTRACT

In this paper, we investigate the pricing problem for a portfolio of life insurance contracts where the life contingent payments are equity-linked depending on the performance of a risky stock or index. The shot-noise effects are incorporated in the modeling of stock prices, implying that sudden jumps in the stock price are allowed, but their effects may gradually decline over time. The contracts are priced using the principle of equivalent utility. Under the assumption of exponential utility, we find the optimal investment strategy and show that the indifference premium solves a non-linear partial integro-differential equation (PIDE). The Feynman–Kač form solutions are derived for two special cases of the PIDE. We further discuss the problem for the asymptotic shot-noise process, and find the probabilistic representation of the indifference premium. We also provide some numerical examples and analyze parameter sensitivities for the results obtained in this paper.

1. Introduction

The principle of equivalent utility (also called the utility indifference principle) has become a popular method for pricing and hedging financial risks in an incomplete market. Young and Zariphopoulou (2002) explored the use of this principle to price insurance risks in a dynamic financial market setting. As in the classical case, the basic idea of this pricing approach is that the valuation of the insurance risks is based on the comparison of the maximal expected utility functions with and without writing the life insurance contracts. The indifference premium (called the reservation price in Young and Zariphopoulou, 2002) is determined by solving the stochastic optimization problem, where the solution is generally obtained by solving the so-called Hamilton–Jacobi–Bellman (HJB) equation. Explicit solutions are normally found when the exponential utility is assumed.

The new mechanism introduced in Young and Zariphopoulou (2002) for pricing dynamic insurance risks enables the study of various problems under a similar financial market environment. Young (2003) applied this principle of equivalent utility to price equity-indexed life insurance and calculate reserves, while Moore and Young (2003) priced equity-indexed pure endowments. Jaimungal and Young (2005) investigated the problem in a financial market driven by a Lévy process. Ludkovski and Young (2008) studied the effects of the stochastic hazard rate and stochastic interest rate. Delong (2009) explored the pricing and hedging problem for a group of life insurance contracts in the presence of systematic mortality risks. Ma and Yu (2006) studied the pricing problem for a class of universal variable life insurance products. Ma et al. (2011) further explored the indifference pricing of a home reversion plan for a pair of insureds.

In financial markets, the price of a risky asset (e.g., a stock) is usually modeled by a geometric Brownian motion, in which the jump of the stock price is incorporated through the jump–diffusion (Merton, 1976). Schmidt and Stute (2007) pointed out that the information would sometimes come as a surprise, which causes a jump (positive or negative) in stock prices, and the effect after the jump would not vanish immediately but instead decline gradually as time passes by until next jump occurs. The market is then incomplete. This phenomenon can be described by adding a shot-noise process to a standard model for stock prices. Generally, researchers often choose the exponential Lévy process to model the asset dynamics in order to capture the heavy-tailed nature of the asset returns. In this paper, we use a shot-noise process with exponential decay to describe jumps and their effects over time and embed it into a classical Black–Scholes model to model the stock prices. Bi and Guo (2010) considered the risk-minimizing hedging problem for unit-linked life insurance in a financial market driven by a shot-noise process. More details on properties and the use of shot-noise processes in financial market can be found in Schmidt and Stute (2007) and Altmann et al. (2008) and the references therein.

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In this paper, we use the mechanism introduced in Young and Zariphopoulou (2002) to investigate the pricing problem for life insurance contracts with equity-indexed life contingent payments. The shot-noise effects are incorporated in the model for the stock prices, which means that sudden jumps in the stock price are allowed but their effects may fade away gradually. We assume that the shot-noise process is exponentially decayed, for which Markovian property holds. We price the contracts through the principle of equivalent utility by comparing the maximal expected utility functions with and without writing the life insurance contracts. By using stochastic control HJB equation method, we provide a detailed analysis for this problem and obtain the HJB equation for general utility functions. Assuming an exponential utility function, we find an optimal investment strategy and show that the indifference premium solves a non-linear PIDE. We consider two special cases of the PIDE; one is when the jump size of the shot-noise process is small, and the other is when the insurer is risk neutral by letting the constant absolute risk aversion coefficient be zero. The Feynman–Kač form solutions (see Björk, 2004) are derived for both cases. We also discuss the indifference premium for the asymptotic shot-noise process by applying the results obtained in Dassios et al. (2005), and find its probabilistic representation.

We intend to show in this paper that the methods and techniques used in Young and Zariphopoulou (2002) and Delong (2009) can also be utilized to solve similar problems under a different financial market condition. In Young and Zariphopoulou (2002) the price of the stock is modeled by a geometric Brownian motion and in Delong (2009) the price dynamics of a risky asset is driven by a Lévy process, while in our paper the stochastic price of a risky asset is described by a Black–Scholes type jump–diffusion model where the jumps are modeled by a shot-noise process. However, as in their papers, we apply a same principle of equivalent utility, and under the assumption of exponential utility, we obtain the optimal investment strategy and show that the indifference premium solves a non-linear PIDE. We point out that we also extend the model studied in Delong (2009), where a life insurance portfolio is considered, in the sense that the payouts for a group of people are equity-indexed in our paper rather than fixed rates as in their paper.

The rest of the paper is organized as follows. In Section 2, we present our model and formulate the problem. In Section 3, by using the stochastic control HJB equation technique, we analyze this problem and derive the corresponding HJB equation. Under the assumption of exponential utility, we find the optimal investment strategy and derive a PIDE satisfied by the indifference premium. In Section 4, we discuss the problem for the asymptotic shot-noise process, and find the probabilistic representation of the indifference premium. In Section 5, we present some numerical examples to illustrate and explore the impact of the jump intensity, risk aversion, and shot-noise effect on the indifference premium price and the Delta hedging parameter, which implies the additional shares of stock that the insurer needs to hold for each policy to hedge the insurance risks.

2. The model

We consider a complete probability space $(\Omega, \mathbb{F}, \mathbb{P})$ with a filtration $\mathbb{F} = (\mathcal{F}_t)_{0\le t\le T}$, where T denotes a finite and fixed time horizon. The filtration $\mathbb{F}$ consists of two subfiltrations: $\mathbb{F} = \mathbb{F}^F \vee \mathbb{F}^N$, where $\mathbb{F}^F$ contains information about the financial market and $\mathbb{F}^N$ contains information about the number of survivors in the portfolio. Suppose that the subfiltrations $\mathbb{F}^F$ and $\mathbb{F}^N$ are independent. Let $\mathbb{E}$ denote the expectation with respect to $\mathbb{P}$.

2.1. The financial market

In this paper, we consider a shot-noise version of a Black–Scholes financial market. The price of a risk-free asset R_t is modeled by $dR_t = rR_t dt$, and the price of a risky asset is described by

$$S_t = S_0 \exp\left\{\mu t + \sigma W_t - \frac{\sigma^2}{2} t + \lambda_t\right\}, \qquad t \ge 0, \tag{2.1}$$

where r, μ, and σ are positive constants such that $\mu > r$. In (2.1), $\{W_t\}_{0\le t\le T}$ is a standard $\mathbb{F}^F$ adapted Brownian motion and $\{\lambda_t\}_{t\ge 0}$ is a shot-noise process defined by

$$\lambda_t = \lambda_0 e^{-\varpi t} + e^{-\varpi t} \sum_{i=1}^{N_t} Y_i e^{\varpi \xi_i}, \qquad t \ge 0,$$

where λ_0 is the initial value of λ_t, $\{Y_i\}_{i=1,2,\ldots}$ is a sequence of independent and identically distributed (i.i.d.) random variables with distribution function $\Psi(\cdot)$, $\mathbb{E}(Y_1) = \mu_1$, and $\mathbb{E}(Y_1^2) = \mu_2$, $\{\xi_i\}_{i=1,2,\ldots}$ is a sequence of random variables representing the event occurrence times of a Poisson process $\{N_t\}_{t\ge 0}$ with constant intensity ρ, and ϖ is the constant rate of exponential decay. The differential form of λ_t is obtained by differentiating λ_t with respect to t as

$$d\lambda_t = -\varpi \lambda_t dt + \int_{\mathbb{R}} yN(dt, dy),$$

where $N(dt, dy)$ is a Poisson random measure with $\mathbb{E}[N(dt, dy)] = \rho\, dt\, \Psi(dy)$, independent of Brownian motion W_t, and $\tilde{N}(dt, dy) := N(dt, dy) - \rho dt \Psi(dy)$ is an $\mathbb{F}^F$ martingale. Applying Itô's formula to S_t given by (2.1), we have

$$dS_t = S_{t-}\left\{\mu dt + \sigma dW_t - \varpi \lambda_t dt + \int_{\mathbb{R}} (e^y - 1) N(dt, dy)\right\}. \tag{2.2}$$

2.2. The life insurance portfolio

In this section, we extend the model studied in Delong (2009) by considering a life insurance portfolio with equity-indexed payouts for a group of m_0 people. We use $T_1, T_2, \ldots, T_{m_0}$ to denote the future lifetimes of the insured people with the same age x at the time of issuing the policies. Suppose that the random variables $T_1, T_2, \ldots, T_{m_0}$ are i.i.d. with the same mortality intensity function $\varsigma_x(\cdot)$ such that

$$P(T_1 > t) = {}_t p_x = e^{-\int_0^t \varsigma_x(s) ds}, \qquad t \ge 0.$$

Assume that there are three types of equity-indexed payments by the insurer: (1) payments payable continuously at rate $c(S_t)$ as long as the insured is alive, which can be life annuity or death benefit if $c(S_t)$ is positive, or premiums if $c(S_t)$ is negative (see Gerber, 1997, Section 5.5), (2) death benefit per policy payment, denoted by $b(t, S_t)$, payable immediately at the time of death of the insured, and (3) terminal survival benefit payment, denoted by $B(T, S_T)$, payable at time T if the insured is still alive at terminal, and the price process of the risky asset $\{S_t\}_{t\ge 0}$ is described by (2.2). Let $M(t)$, for $0 \le t \le T$, denote the number of survivors in the portfolio at time t, that is,

$$M(t) = m_0 - \sum_{i=1}^{m_0} 1_{\{T_i \le t\}}.$$

Then the dynamics of the insurer's cumulative payment process $Q(t)$, for $0 \le t \le T$, can be described by

$$dQ(t) = M(t-)c(S_t)dt - b(t, S_t)dM(t) + M(T)B(T, S_T)d1_{\{t=T\}},$$

with initial premium $Q(0) = -m_0 P(0)$, where $P(0)$ is the net single premium paid by each insured at time zero when the policy is issued, and $1_{\{\cdot\}}$ is the indicate function.