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Probabilistic models of income distributions

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Abstract

The results showing relationships between distributions of individual incomes and incomes of two-earners households are presented. It is shown that individual incomes are very well described by Dagum's distributions. Income distributions in two quite different countries (USA and Poland) are studied for comparison. Obtained results show very striking and interesting differences.

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1. Introduction

The preliminary results of income distributions of households in two different countries (USA and Poland) are presented. We focus on individual incomes (incomes of one-earner households) and on incomes of two-earners households. Obviously the income of a two-earners household is just the sum of individual incomes. The aim of this work is to examine if there is any relationship between a distribution of individual

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incomes and a distribution of incomes in two-earners households. We present result that concern two countries: USA and Poland. In the case of the USA such a relationship has recently been investigated in Ref. [1]. For a description of individual incomes the authors of that paper used the exponential model. In this work we use another model of income distributions which seems to provide a better fit to experimental data, especially for Polish households. Full results of our investigations relating individual distributions of incomes in the USA can be found at an internet site [2]. They include all the years 1975–2002. Due to the lack of space, in this paper we restrict ourselves to the year 2000. Results concerning distributions of incomes of Polish households for the year 2000 are an extension of our previous paper [3].

2. Models of individual income distributions

To indicate the best possible model for distributions of individual incomes in the USA, we start with four candidates: exponential model, Weibull's model, Dagum's model and Singh–Maddala's model. In the case of individual incomes in Poland we have studied the adjustment of Dagum's, Singh–Maddala's, gamma and log-normal models [3]. The level of adjustment of the last two models turned out to be very low and we do not consider them in this paper. The probability density functions of examined models can be described by the following formulas:

- Dagum's model:

$$f_D(x) = \frac{abcx^{b+1}}{(1+ax^{-b})^{c+1}}, \quad a>1,\ b>0,\ c>0 .$$

- Singh–Maddala's model:

$$f_{SM}(x) = \frac{abcx^{b-1}}{(1+ax^{b})^{c+1}}, \quad a>0,\ b>0,\ c>0,\ bc>1 .$$

- Exponential model:

$$f_E(x) = \frac{1}{a}\exp\left(-\frac{x}{a}\right), \quad a>0 .$$

- Weibull's model:

$$f_W(x) = \frac{b}{a}\,x^{b-1}\exp\left(-\frac{x^b}{a}\right), \quad a>0,\ b>0 .$$

Data of American households are taken from an internet site (file *mar00supp*) [4]. In the case of Polish households the data come from the Main Statistical Office (GUS) in Warsaw, Poland.