A note on joint estimation of scale economies and productivity growth parameters

Efthymios G. Tsionas[a],*, John Loizides[b]

[a]*Council of Economic Advisers, Ministry of National Economy, 5 Nikis Street, Constitution Square, 10180 Athens, Greece*
[b]*Department of International and European Economic Studies, Athens University of Economics and Business, Athens, Greece*

Abstract

In applied research it is common to estimate complete production models, obtain measures of scale economies and total factor productivity growth and then regress such measures on exogenous variables. Such procedures result in inconsistent estimates of technological as well as regression parameters: estimation of total factor productivity and scale economies in the first step, does not take into account that these measures depend on exogenous variables in the second step. Therefore, their dependence on exogenous variables is not properly taken into account in the first step. The study proposes to estimate jointly the cost function, the share equations as well as total factor productivity and scale economies measures, using full system estimation to account for all the restrictions implied by their endogeneity. The approach is illustrated using data from British, French, and German railways. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

In recent years there has been an explosion of interest in estimating technical characteristics of production and cost structure, using complete production models. The purpose of formulating and estimating complete econometric models of production is to assess the extent of substitutability of certain factors of production and evaluate the magnitude of scale economies and productivity growth.[1] However, in many situations of interest researchers believe that scale economies and pro-

ductivity growth have been affected by certain exogenous variables, and then they seek to test empirically whether or not this is the case. To that end, the applied researcher starts with a cost function

$$C(p_t; y_t, t) = C(p_{1t}, p_{2t}, \ldots, p_{nt}, y, t), \quad t = 1, \ldots, T, \quad (1)$$

where p_{it} denotes the price of input i at date t, y_t denotes output and t denotes technological progress. Using standard duality theory [9,10], one can derive input demand functions of the form

$$x_{it} \equiv x_i(p_t, y_t, t) = \frac{\partial C(p_t, y_t, t)}{\partial p_{it}}, \quad i = 1, \ldots, n. \quad (2)$$

---

* Corresponding author. *Present address*: Department of Economics, Athens University of Economics and Business, 104 34 Athens, Greece.

E-mail address: tsionas@aueb.gr (E.G. Tsionas).

[1] See for example [1–8]

Scale economies (SE) and total factor productivity growth (TFP) are obtained as

$$\mathrm{SE}_t = \left(\frac{\partial C(p_t, y_t, t)}{\partial y_t} \frac{y_t}{C(p_t, y_t, t)} \right)^{-1}$$

$$= \frac{\partial \ln C(p_t, y_t, t)}{\partial \ln (y_t)} \tag{3}$$

and

$$\mathrm{TFP}_t = -\frac{\partial C y_t}{\partial t} \frac{1}{y_t} = \frac{\partial \ln y_t}{\partial t}. \tag{4}$$

It is common to estimate the complete system composed of (1) and (2) using FIML or iterative Zellner's technique (IZEF), obtain estimates of SE_t and TFP_t at the final estimates and then regress them on certain exogenous variables z_t:

$$\widehat{\mathrm{SE}}_t = z_t'\alpha + v_{1t},$$

$$\widehat{\mathrm{TFP}}_t = z_t'\beta + v_{2t}. \tag{5}$$

In (5), α and β denote coefficient estimates, v_{1t}, v_{2t} denote error terms and a hat denotes evaluation at the final IZEF estimates. A popular way to estimate the practical importance of certain quantitative and qualitative variables on the level of technical efficiency is to estimate first a technical efficiency index from the residuals of a production function, which includes as exogenous variables some basic inputs of production[2] (capital, labor, energy, land, fertilizer, etc., see for example [11–13] among others) and second to regress technical efficiency or productivity growth on such variables as education, farm assets, credit, etc. The argument has been advanced (e.g. [14]) that although it would be more meaningful to include such variables as formal inputs in the production function, this would be undesirable because it raises some conceptual problems regarding substitutability: For example, it makes no sense to argue that there is substitution between fertilizer and education – in an agricultural economics context, or between average trip length and labor – in a railway economics context. For that reason, there is consider-

able incentive for defending the "regression approach". However, there is little reason to support the two-step approach that is commonly used in the literature. In fact, the two-step approach suffers from several problems:

(i) Regressions like (5) ignore parameter uncertainty and simply compute the dependent variable at the final estimates, without taking proper account of their variability. As a matter of fact, these regressions ignore the fact that TFP or SE have been computed in the first round, under the assumption that they do not depend on exogenous variables, i.e. that they are strictly exogenous variables. In the second round, however, this assumption is (silently) violated.

(ii) Regressions like (5) are problematic because they introduce additional variables z_t into the problem in an ad hoc manner. Formally, variables in z_t should enter (1) as inputs, however, in that case one would have substitution between z_t and the existing inputs (unless variables in z_t enter as fixed factors in the short run). This may not always be a reasonable approach to the problem. Also, (5) is problematic because it does not have a direct interpretation.

(iii) Regressions (5) in conjunction with (3) and (4) suggest that prices or output are endogenous and should depend on z_t. If that is the case, joint estimation of (1) and (2) by IZEF will yield inconsistent results, and the same is true for OLS applied to (5).

A way out of these problems is to estimate (1) and (2) jointly with (3) and (4). In general, (3) and (4) are functions of prices, output, time and the structural parameters of the cost function, so joint estimation introduces additional restrictions (on top of homogeneity, symmetry, etc.) on structural parameters imposed by the endogeneity of TFP and SE.

2. Application to the translog technology

To fix ideas, assume that the cost function can be described by the generalized translog technology, in which case it follows that

$$\ln C(p; y) = \alpha_0 + \alpha_y \ln y + \tfrac{1}{2}\alpha_{yy}(\ln y)^2$$

$$+ \sum_{i=1}^{n} \beta_i \ln p_i + \tfrac{1}{2}\sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j$$

---

[2] The problem arises in many fields of applied economics, but it received a great deal of attention in agricultural economics.