Chaos, Solitons and Fractals 103 (2017) 289–293

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Probabilistic reward or punishment promotes cooperation in evolutionary games

Yu'e Wu [a], Bin Zhang [b], Shuhua Zhang [a,*]

[a] Coordinated Innovation Center for Computable Modeling in Management Science, Tianjin University of Finance and Economics, Tianjin 300222, China
[b] Beijing Institute of Satellite Information Engineering, Beijing 100086, China

ARTICLE INFO

Article history:
Received 8 May 2017
Revised 7 June 2017
Accepted 17 June 2017

Keywords:
Prisoner's dilemma
Reward
Punishment
Self-interaction
Complex network

ABSTRACT

Reward and punishment are crucial for the emergence and sustainability of cooperation in evolutionary games. In this work, we introduce a third-party agent, who plays the role of a judge to reward cooperators and punish defectors, to study the impact of reward and punishment on the evolution of cooperation. The introduced righteous agent is different from the cooperators and defectors in the traditional games and it exists as a judge independent of the processes of the games. In each round of the evolutionary game, each player has a half chance to confront the righteous agent. If the player is a cooperator, it's possible for it to obtain an extra profit. On the contrary, when a defector meets the righteous agent, its earnings may be reduced. The simulation results show that the introduction of the righteous agent in the evolutionary game favors the evolution of cooperation. The robustness of the promoting effect is tested for different complex topologies for the prisoner's dilemma game. The enhancement effects are confirmed in the snowdrift game as well, which may imply that the facilitation effects show a high degree of universality independent of the structure of the applied spatial networks and the potential evolutionary game models. Our conclusion may be conducive to interpret the emergence and sustainability of cooperation within the structured populations.

1. Introduction

The emergence and maintenance of cooperation among unrelated individuals is a ubiquitous phenomenon in both biological and social systems [1–5], which is inconsistent with the description in *Darwin's The Origin of Species*. For elucidating this puzzle, the evolutionary game theory is adopted as a common formal framework to study the evolution of cooperation in social dilemmas, in which social contradictions are analogous to the competition among peers for limited resources [6–9].

A variety of game models including the prisoner's dilemma game (PDG) and the snowdrift game (SDG) are proposed to investigate the evolution of cooperation [10–15]. In the traditional games, two involved players are asked to make a choice (cooperate or defect) simultaneously. If they both choose to cooperate, they will obtain the highest collective payoff and receive a reward R, respectively. On the other hand, mutual defection yields the lowest collective payoff and each of the two encountering defectors obtains a punishment P. If an agent chooses to defect while the opponent cooperates, the defector receives a temptation to defect T, and the cooperator obtains a sucker's payoff S. For the PDG, the ranking of these four payoffs is $T > R > P > S$, which implies that the best strategy for each agent is to defect regardless of the opponent's decision. The payoff ranking is inconsistent with the fact that cooperative behaviors are widely observed in nature [16–18]. In fact, the largest individual interests do not bring the greatest collective interests, which is precisely the so-called social dilemma. For the SDG, players interact in a similar way, but the payoff ranking is $T > R > S > P$. This minor variation induces a significant change in the game dynamics with the creation of a second Nash equilibrium where the optimal strategy for the player is the opposite of the opponent's (defect when your opponent cooperates and cooperate when your opponent defects) [10,11,19,20].

In the past few years, a large number of scenarios have been proposed to understand the origin and evolution of cooperation [3,21–23]. Examples include asymmetry of learning and teaching activities [24–26], personal reputation [27,28], different update rules [3,29,30], reward and punishment [31–35]. These mechanisms have been attributed to five aspects by Nowark: kin selection, direction reciprocity, indirect reciprocity, group selection and network reciprocity [36]. Among these mechanisms, network reciprocity, where agents are arranged on the spatially structured topology and play only with their direct neighbors, is a well-

* Corresponding author.
E-mail address: szhang@fjufe.edu.cn (S. Zhang).

http://dx.doi.org/10.1016/j.chaos.2017.06.015
0960-0779/

known dynamical rule that favors the evolution of cooperation [37–40]. Because cooperators can form compact clusters which minimizes the aggression by defectors and protects the agents that are situated in the interior of such clusters. The evolution of cooperation on networks has been extensively explored in a large number of network structures since Nowak and May proposed the first model of the networking PDG, where the agents were populated on the square lattices [41]. Meanwhile, even in the same network topology structure, numerous evolutionary mechanisms are proposed to understand the prevalence of cooperation, such as the mechanism mentioned above [42–44].

Of particular renown, the impact of the reward and punishment on the evolution of cooperation has received great attention till now. In vast majority of existing works, most of the authors focus their attentions either on the role of reward or on the role of punishment [32,33,45–47]. Besides, in most these papers, cooperators (defectors) receive rewards (punishments) as a second-stage behavior and the game models are almost limited to public goods game [31,48–55]. There is a situation of particular relevance that has received relatively little attention. This is the case of combination of the reward and punishment in one mechanism. In the present paper, we consider a special righteous agent who will play the role of a reward agent and punisher simultaneously in the PDG on the square lattice. Each player in the game has a half chance to confront the 'righteous guy'. If the player is a cooperator, it will obtain an additional bonus; on the contrary, a defector will be fined. Different classes of networks including Erdös–Rényi (ER) graph and Barabási–Albert (BA) scale-free (SF) network are considered in the spatial PDG as well. The robustness of the simulation results are tested in the SDG on the square lattice. The simulation results show that the introduction of the righteous agent can improve the evolution of cooperation dramatically.

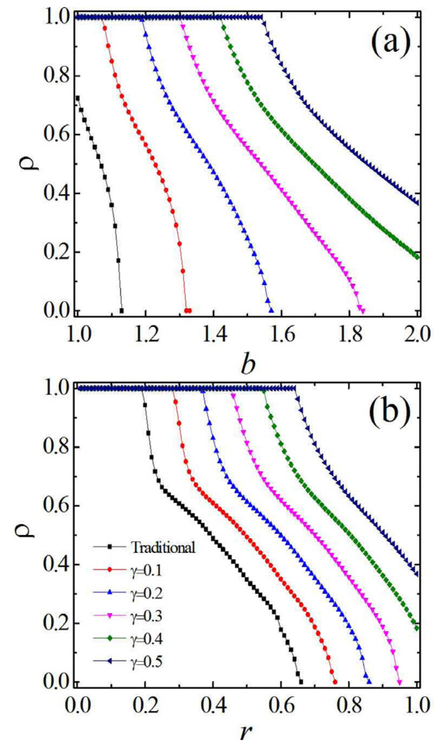

Fig. 1. The fraction of cooperators (ρ) in dependence on b (prisoner's dilemma game) and r (snowdrift game) for different values of γ. Compared with traditional version (namely, $\gamma = 0$), it is obvious that the introduced mechanism promotes cooperation. All the results are obtained in the von Neumann neighborhood on the square lattice for $L = 100$, MCS = 60,000 and $k = 4$.

2. Method

We consider an evolutionary two-strategy PDG with agents populated on the vertices of the discussed networks. Following a common parametrization, the payoffs in the PDG are set as follows: $T = b > 1$, $R=1$ and $P = S = 0$ satisfying the ranking $T > R > P \geq S$, which captures the essential social dilemma between individual and common interests. It is worth noting that even if we choose a weak and simple version (namely, $S = P = 0$), the simulation results are robust and can be observed in the full parameterized space. For the SDG, a similar mechanism with $T=1+r$, $R=1$, $S=1-r$ and $P=0$ is adopted, where $0< r \leq 1$ represents the so-called cost-to-benefit ration for satisfying the payoff ranking $T > R > S > P$.

In this work, we introduce a third-party agent in the 2-person evolutionary games (PDG and SDG), in which the introduced agent plays the role of a judge. In the model, the agents (the cooperators and the defectors) in the traditional evolutionary game have a half chance to confront the third-party agent. If a cooperator encounters the third-party agent, it will obtain an additional benefit γ for its selfless behavior. Meanwhile, if a defector meets the third-part agent, it will be punished for its selfishness and its payoff will be reduced by γ. The value of γ is changed from 0.0 to 0.5. When $\gamma = 0$, the model is reduced to the standard spatial PDG or SDG and no additional reward or punishment is applied to the players. The robustness of the results is tested by considering three different classes of networks including regular square lattices with periodic boundary conditions, Erdös–Rényi random graphs (ER) and Barabási–Albert scale-free (SF) networks.

We implement the evolutionary dynamics in the following elementary steps. In the initial stage, both the cooperators (C) and the defectors (D) have the same probability to occupy the vertices of the network topology structure. Then, each player i in the network plays with all its neighbors at each time step, and gets a payoff P_i by adding all the obtained payoffs. Next, all the agents synchronously update their strategies employing the finite populations analogue of replicator dynamics. The focal player i updates its strategy by picking up one of its neighbors at random, say j, and comparing the respective payoffs P_i and P_j. If $P_i > P_j$, agent i will keep its strategy for the next step. On the contrary, if $P_i < P_j$, player i will copy the neighbor's strategy with a probability proportional to the payoff difference:

$$W_{i \longleftarrow j} = \frac{P_j - P_i}{\max\{k_i, k_j\}d}, \tag{1}$$

where k_i and k_j represent the degrees of players i and j, respectively, and d stands for the maximum possible payoff difference between two agents ($d = b$ for the PDG and $d = 1 + r$ for the SDG). From the equation, it is not difficult to predict that it is possible for each player to shift from one strategy to another.

The key quantity the fraction of cooperators ρ is determined by averaging the last 10^4 full MCS (Monte Carlo Simulation) over the total 6×10^5 steps. All the results are averaged over 40 independent rounds. In addition, the size N and the average degree $< k >$ of all the considered networks are set as $N = 10^4$ nodes and $< k > = 4$, respectively.

3. Results and analysis

We firstly depict the fraction of cooperation (ρ) on a square lattice with periodic boundary conditions at the stationary state in Fig. 1. Panel (a) and panel (b) represent the results in the PDG and the SDG, respectively. Each agent plays the game with its four nearest neighbors (von Neumann neighborhood) and a reward or a fine γ may be imposed on the agent according to its strategy. If the focal player chooses cooperation strategy, it may acquire an ex-