A metric for assessing the "goodness" of income distributions and the effect of price changes

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Abstract

Social welfare evaluation depends in part on value judgments as to income distribution. This paper proposes a metric for assessing the "goodness" of particular income distributions. That metric is then used to examine the effect of price changes on the "goodness" of a given distribution. Consider an increase in the price of a commodity that is disproportionately consumed by households with incomes that are high relative to the preferred income distribution. One naturally supposes that such a price increase will make the given income distribution appear less bad. Surprisingly, this is not invariably the case.
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1. Introduction

As is well known, social welfare evaluation of different allocations of commodities to individuals depends in part on value judgments as to income distribution. In this paper, I propose a metric for assessing how good or bad a particular income distribution is thought to be. I then go on to use that metric to examine the effect of price changes on the "goodness" of a given distribution of money income. Consider an increase in the price of a commodity that is disproportionately consumed by households with incomes that are high relative to the preferred income distribution.

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Since what matters to welfare and to welfare judgments is real, not money income, one naturally supposes that such a price increase will make the income distribution under consideration less bad. Surprisingly, this is not invariably the case.

2. Defining the metric

There are n commodities and H households. The consumption of commodity i by household h is denoted by x_i^h ($i = 1, \ldots, n$; $h = 1, \ldots, H$). The vector of h's consumption is denoted by x^h. y^h denotes h's money income.

Total consumption of commodity i is denoted by $X_i = \sum_{h=1}^{H} x_i^h$ and total money income by $Y = \sum_{h=1}^{H} y^h$. The vector of total consumption is denoted by X.

The price of commodity i is denoted by p_i. The vector of prices is denoted by p, and pX is the inner product of p and X—the total expenditure over all households and all commodities.

Household h's utility function is denoted by $u^h = u^h(x^h)$, and u denotes the vector of the u^h.

The social welfare function is

$$W = W(u^1, \ldots, u^n) = W(u). \tag{2.1}$$

Parallel to the definition of the expenditure function in the theory of the individual household, we can define a social welfare expenditure function (WEF). In its unrestricted form, that function is defined as

$$E^* = E^*(p, W) = \operatorname{Min} pX, \quad \text{subject to } W(u) = W. \tag{2.2}$$

In other words, E^* is the minimum total money income needed by the household sector to attain social welfare W when prices are p, provided that such total income can be allocated by the social planner whose welfare function is $W(\cdot)$.[1]

But, of course (and, perhaps, fortunately), social planners typically do not have such powers, and this will increase the cost of obtaining welfare level, W. More generally, the social planner is, for purposes of this paper, a fictitious construction. What we are actually interested in here is the use of that fiction to define a metric of the "goodness" or "badness" of the existing income distribution as seen from the viewpoint of someone making value judgments about it.

In considering social welfare, one can contemplate both changing the income distribution and changing the level of total income. From the point of view of a

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[1] It has not escaped my attention (and could not possibly do so when writing a paper in honor of Karl Shell) that the social welfare expenditure function can be used to create a social welfare price index in the form of $E^*(p^1, W)/E^*(p^0, W)$, where p^0 and p^1 are the price vectors to be compared. That index would compare the relative cost of achieving a given level of social welfare at the two price vectors. (And the same could be done for the restricted WEF about to be defined.) The theory of such an index would obviously be isomorphic to that of the true cost-of-living price index (see, for example, [3, Essay I]). At least for the present, however, I do not see that social welfare functions can typically be specified sufficiently closely to permit the deriving of useful theorems here, and my exploration of the effect of price changes (below) takes a somewhat different road.