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Size-controllable self-imaging of one-dimensional fractal grating

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ABSTRACT

Fresnel diffraction of one-dimensional curved fractal grating is studied, and the reduced, identified and magnified Talbot images of fractal grating are obtained by the proposed optical system. Two kinds of fractal gratings formed by adding and multiplying periodic gratings are chosen as examples. The analytic expressions of the Fresnel diffraction for one-dimensional fractal gratings are deduced, and the size-controllable self-images are predicted to appear at the modified Talbot distance. The longitudinal and transverse diffraction intensity distributions of the curved fractal gratings are simulated. The uniformity of the longitudinal diffraction carpets can be controlled by changing the system parameters, and the transverse diffraction distributions take on the compressed, exact, and magnified images of fractal grating. These size-controllable Talbot imaging of fractal grating may extend the applications of fractal grating because of the additional flexibility.

1. Introduction

Self-imaging of grating is an important optical phenomenon of periodic object in Fresnel diffraction. Since it was advanced by Talbot W H F [1], this phenomenon has been developed [2,3] and the studies about the self-imaging of the periodic and quasi-periodic structures have been paid much attentions [4–6]. Importantly, the self-imaging phenomenon has also been widely used in many fields such as array illumination [7], pulse shaping [8] and optical lithography [9]. However, the significance of Talbot effect is not only for some optical phenomena, but also for the expanding regulations in wide science field, such as X-ray [10], quantum physics [11], and nonlinear effects [12]. Also, Talbot effect is extended into a more complex geometry, such as fractal objects. Fractals, such as Cantor sets and Koch curves, have the unique self-similarity and scale-invariant property [13]. With these distinctive characteristics, the fractal grating which combines grating with fractal structure has shown unique advantages in resonator transmission [14] and optical filtering [15]. Therefore, the self-imaging effect of fractal grating arouses the interest of researchers [16–19].

In previous researches, including our studies about Talbot effect of periodic grating [20–22], quasi-periodic grating [6], and fractal grating [18,19], the grating is straight and the presented Talbot carpet is homogeneous. Thus, only the identified (integral) and reduced (fractional) images of grating are obtained. In fact, the curvature of grating and the divergence of light source may cause the image of grating to magnify [23,24]. Recently, our study about the diffraction of curved grating shows that the magnified image of periodic grating can be formed in Fresnel diffraction region [23]. In this paper, we concentrate

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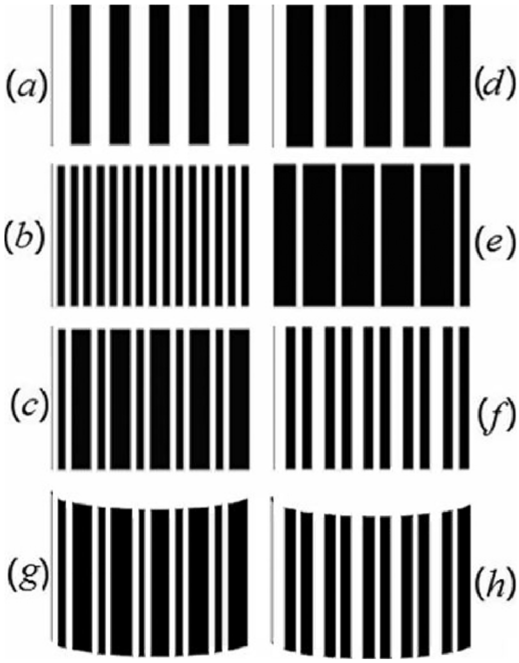

Fig. 1. Elementary gratings (*a,b,d,e*), the first kind of fractal grating (*c*) and the second kind of fractal grating (*f*), and curved fractal gratings (*g,h*).

our study on Fresnel diffraction of the curved fractal grating, and aim to obtain the size-controllable self-image of fractal grating. The organization of the content is as follow. Section 2 gives the description of two kinds of the curved fractal gratings. Section 3 presents the theoretical analysis about the diffractions of the curved fractal gratings and predicts the Talbot images of fractal gratings with different size appear at the modified Talbot distances. Section 4 shows the simulations for the diffractions of the curved fractal gratings and the unequal images of fractal gratings verify the artificial manipulation of Talbot imaging of grating. In the end, the conclusions of this paper are provided.

2. Curved fractal gratings

Here, the fractal grating contains two cases, and we name them as the first and second kind of fractal gratings, respectively, like in our previous researches [19]. The first kind of fractal grating is formed by multiplying elementary periodic gratings and the second kind of fractal grating is generated by adding elementary periodic gratings. As we know, the transmission function of the periodic grating can be expressed in the Fourier series [25]. The elementary gratings for the first kind of fractal grating can be expressed by $\Sigma_m A_m \exp(i2\pi m x_0/d_i)$ with A_m representing Fourier coefficient and d_i denoting the period of grating. Similarly, the elementary gratings for the second kind of fractal grating can be expressed by $\Sigma_m A_m \exp(i2\pi m(x_0\text{-}a_i)/d)$, where d is the period of grating and a_i is the spatial dislocation.

Fig. 1 shows two fractal grating samples to illuminate the formation process of the curved fractal grating. Fig. 1(*a*) and (*b*) are two elementary gratings for the first kind of fractal grating. The opening ratios of two gratings are 1/2 and the period of one grating is three times of the other one. Fig. 1(*c*) is the first kind of fractal grating by multiplying the gratings in Fig. 1(*a*) and (*b*). This fractal grating is 1-level fractal and its dimension is $D = 0.6309$. Similarly, Fig. 1(*d*) and (e) show two elementary gratings for the second kind of fractal grating. The periods of two gratings are the same and their opening ratio 1/3 and 1/6, respectively. The spatial dislocation between two gratings is half of period. Fig. 1(*f*) is the second kind of fractal grating by adding the gratings in Fig. 1(*d*) and (*e*). This fractal grating is also 1-level fractal and its dimension is $D = 1$. Next, bending the fractal gratings along a cylindrical surface, we can obtain the curved fractal gratings, as shown in Fig. 1(*g*) and (*h*).

The transmission functions of the two kinds of one-dimensional fractal gratings, as shown in Fig. 1(*c*) and (*f*), can be expressed by,

$$t_1(x_0) = \sum_m \sum_n A_m A_n \exp(\frac{i2\pi m x_0}{d_1})\exp(\frac{i2\pi n x_0}{d_2}) \tag{1}$$

and

$$t_2(x_0) = \sum_m A_m \exp(\frac{i2\pi m x_0}{d}) + \sum_n A_n \exp[\frac{i2\pi n(x_0 - a)}{d}] \tag{2}$$