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Admissibility analysis for discrete-time singular Markov jump systems with asynchronous switching

Xianwen Gao [a], Hangfeng He [a,*], Wenhai Qi [b]

[a] College of Information Science and Engineering, Northeastern University, Shenyang 110819, China
[b] Department of Automation, Qufu Normal University, Rizhao 276826, Shandong, China

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ABSTRACT

This paper deals with admissibility analysis for discrete-time singular Markov jump systems (SMJSs) with asynchronous switching, which means the switching of controllers is asynchronous with the switching of subsystems. The switching delay and state delay are modeled as Markov chains. The resulting closed-loop system is modeled as a singular system with a Markovian jumping vector and two Markovian jumping parameters. A new necessary and sufficient condition for regularity, causality and stability of the closed-loop system is proposed. Based on this, a state feedback controller is designed in terms of a strict linear matrix inequality (LMI) to guarantee the admissibility of SMJSs with asynchronous switching. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

In the past decades, Markov jump systems have been a subject of great practical importance over the past few decades. The systems can effectively model dynamical processes involved with stochastic switching subject to a Markov chain, such as networked control systems [1–3] and fault-detection systems [4]. As Markov jump systems have drawn much attention, there are many achievements on Markov jump systems [5,6]. The stabilization of Markov jump nonlinear systems has been investigated in [7–9]. The H_∞ observer has been designed in [10] for stochastic time-delayed Markov jump systems under partly known transition rates and actuator saturation. In a more general case, Markov jump systems with complex transition probabilities have been analyzed in [11,12].

At the same time, singular systems are also known as generalized state-space systems in engineering field, such as engineering systems, chemical processes, economic systems, communication networks, etc.; see [13,14]. Many results have been reported on admissibility analysis [15], output feedback control [16], reachable set estimation [17], mode-dependent quantize H_∞ filtering [18], soft variable structure control [19] and fault tolerant control [20]. Recently, a great deal of attention has been devoted to singular Markov jump systems. To mention a few, necessary and sufficient conditions for the stochastic stability of SMJSs [21] have been given. Considering time delays, stability and stabilization [22], extended passive filtering [23], delay-dependent bounded real lemmas [24] for SMJSs have been investigated. However, the transition probabilities may be partly unknown. When taking partly unknown transition probabilities into account, state feedback controller [25,26] and anti-windup compensator [27] have been designed.

* Corresponding author.
E-mail address: hehangfengedu@163.com (H. He).

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On the other hand, time delay is inherent in many physical systems, which may cause instability and undesirable performance. However, most of the researches are focused on time delay in the states [28–30]. When the delay occurs in the switching signal, there may be a mismatch between controllers and subsystems. Although many slowly switched systems with asynchronous switching have been investigated [31–34], a few researches about asynchronous Markov jump systems have reported [35–38]. The issue of asynchronous passive control has been addressed for Markov jump systems in [35] without switching delay, in which the asynchronous control caused by the uncertainty of transition probability. Asynchronous filtering for Markov jump systems has been investigated in [36,37]. The stabilization problem for a class of discrete-time Markov jump linear systems with time-delays both in the system state and in the mode signal has been addressed in [39]. But the delay in the switching signal was modeled as a constant. In [40], the random communication delay has been considered in the switching signal.

Above all, the problem of singular Markov jump systems with asynchronous switching is important in both theory and practice. Although the importance of asynchronous switching has been widely recognized, few related results have been established for singular Markov jump systems. To the best of our knowledge, there is no result on discrete-time singular Markov jump systems with asynchronous switching presented in existing literatures, which motivates our study. The main contribution of this paper lies in three aspects: (i) By modeling the closed-loop system as a singular system with a Markovian jumping vector and two Markovian jumping parameters, the necessary and sufficient condition of the admissibility is proposed; (ii) A state feedback controller is designed to guarantee the admissibility of the closed-loop systems; (iii) Ignoring the state delay, necessary and sufficient condition for the design of controller is proposed.

The rest of this paper is outlined as follows. In Section 2, discrete-time singular Markov jump systems with asynchronous switching is introduced and the closed-loop system is modeled as a singular system with a Markovian jumping vector and two Markovian jumping parameters. In Section 3, the necessary and sufficient condition of the admissibility is proposed and a state feedback controller is designed. Section 4 gives a numerical example, followed by conclusion in Section 5.

2. Problem formulation

Fix the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider a class of system with Markov jumping parameters as follows:

$$Ex(k+1) = A(r(k))x(k) + B(r(k))u(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ is the control input. $E \in \mathbb{R}^{n\times n}$ is singular with $rankE = \varepsilon < n$, $A(r(k)) \in \mathbb{R}^{n\times n}$, $B(r(k)) \in \mathbb{R}^{n\times m}$ are known real matrices. The mode jumping process belongs to $\mathcal{N} = \{1, 2, \cdots, \eta\}$ with the transition probability matrix $\Pi_1 = \{\pi_{ij}\}$:

$$Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij}, \tag{2}$$

for all $i, j \in \mathcal{N}$, $\pi_{ij} \in [0, 1]$, and for all $i \in \mathcal{N}$, $\sum_{j=1}^{N} \pi_{ij} = 1$.

Considering the time delay in switching signal and state, we have the controller as follows:

$$u(k) = K(r(k - \tau_c(k)))x(k - \tau_s(k)), \tag{3}$$

where τ_c is modeled as a homogeneous Markov chain $S = \{0, 1, 2, \dots, d_c\}$ and τ_s is modeled as a homogeneous Markov chain $\Lambda = \{0, 1, 2, \dots, d_s\}$. The related transition probability matrices are defined $\Pi_2 = \{\lambda_{gf}\}$ and $\Pi_3 = \{\theta_{hl}\}$ with the probabilities:

$$\begin{aligned} \lambda_{gf} &= Pr\{\tau_c(k+1) = f | \tau_c(k) = g\}, g, f \in S, \\ \theta_{hl} &= Pr\{\tau_s(k+1) = l | \tau_s(k) = h\}, h, l \in \Lambda. \end{aligned} \tag{4}$$

Considering system (1) and the asynchronous controller (3), the resulting closed-loop system is given by

$$Ex(k+1) = A(r(k))x(k) + B(r(k))K(r(k - \tau_c(k)))x(k - \tau_s(k)). \tag{5}$$

Remark 1. Different from the Markovian jump systems with synchronous switching, the controller in this paper is asynchronous with subsystem. In [5]–[12], the mode of controller and the mode of subsystem always stay the same. With the delay in switching signal, the controller in this paper may be unmatched with subsystem, which brings difficulties for analysis.

Remark 2. It is noted that when $\tau_c(k)$ is a constant, the asynchronous controller (3) reduces to the controller in [39], and when $\tau_c(k) = \tau_s(k)$, the controller (3) reduces to the controller in [40]. Thus, the asynchronous controller given in this paper covers the controllers in [39] and [40] as special cases. Moreover, the system (5) in this paper is singular.

Refer to the network-based Markov jump system [40], we define:

$$\begin{aligned} \xi(k) &= \begin{bmatrix} x^T(k) & x^T(k-1) & x^T(k-2) & \cdots & x^T(k-d_s) \end{bmatrix}^T, \\ \hat{r}(k) &= \begin{bmatrix} r(k) & r(k-1) & \cdots & r(k-\tau_c(k)) & \cdots & r(k-d_c) \end{bmatrix}^T, \\ \hat{K}(\hat{r}(k)) &= \begin{bmatrix} K^T(r(k)) & \cdots & K^T(r(k-\tau_c(k))) & \cdots & K^T(r(k-d_c)) \end{bmatrix}^T. \end{aligned} \tag{6}$$