Fuzzy arithmetical stability analysis of uncertain machining systems

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ABSTRACT

The dynamical behavior of machining processes with parameter uncertainties is analyzed using the possibilistic approach of fuzzy arithmetic. The concept is used to simulate and analyze uncertain models of different complexity. Emphasis is put on the analysis of dynamic stability and the inclusion of the uncertain parameters therein. The stability limits, which are defined by an implicit equation, yield two-dimensional fuzzy-valued results, so that standard solution methods are not directly applicable. Therefore, a general method for the solution of implicit equations is presented that may be applied to the stability analysis of arbitrary time-delayed systems. The presented method as well as a fuzzy sensitivity analysis are then applied to the stability analysis of exemplary systems. The propagation characteristic as well as the fuzzy sensitivity analysis allow to see the effect of uncertainties on the system output and quantify the effect of given uncertain parameters.

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1. Introduction

Machining processes belong to the key production steps in almost any modern industry. Operations like turning, milling, boring, or grinding help to manufacture all kinds of products, ranging from heavy machinery to lightweight mobile phones. The time necessary for processing the wrought materials is a serious expense factor. A principal demand on any machining process is therefore to be as short as possible. Reducing cycle times yields increased cutting speeds and higher material removal rates. Higher material removal rates in turn may induce vibrations in the system and thus lead to increased tool wear and a loss of overall machining quality.

Scientific investigation of machining processes and the underlying dynamics consequently play an important role, e.g. improving tools, machines, or the process itself, [1]. Stability analysis methods allow to characterize the dynamic properties of the machining systems and help the operator to choose the process parameters right to achieve low cycle rates without loss of quality.

The researcher working on machining problems faces numerous modeling challenges. It is one of the main problems that parameters and properties of the processes are often not exactly known. Notably the material removal is a very complex process that involves a multitude of different effects.

Stability lobe diagrams with uncertainties have attained significant attention in recent years. Therefore, researchers have chosen different types of methods to handle uncertainty analysis for the calculation of stability lobe diagrams. Of course, probabilistic theory is applied, [2,3], for which the quantification of the parameter distributions is difficult. The edge theorem

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and linear matrix inequality is applied, [4], further, a spectral radius is applied, [5,6]. However, there are several assumptions necessary, such as small uncertainty or time-invariant systems. An optimization approach for the calculation of an upper and lower bound is presented in [7], and an approach for the calculation of the lower bound utilizing the manifold of the stability limit is presented in [8].

In this work, fuzzy arithmetic, [9,10,11], will be employed to analyze and investigate the influence of the uncertainties on stability charts of time-delayed systems. The uncertainty, which is present in the system, is quantified with the help of fuzzy parameters. The propagation of the uncertainty, i.e. the simulation including the fuzzy parameters, yields fuzzy-valued results that reflect the influence of uncertainty on the output of the system. Similar analysis has been conducted in [12,13] with standard fuzzy arithmetical solution procedures. However, the standard solution procedures, which are applied in those publications, are not generally applicable for the stability analysis because of two facts. Firstly, interval arithmetic fails due to the non-monotonicity of the investigated system. And secondly, the transformation method requires a discretization of the domain of the stability lobe diagram for this application, as will be shown in this paper. The way of application of the transformation method in [13] succeeds, albeit the theoretical basics for this manner of solution scheme and the justification for the application is provided in this work.

Fuzzified stability limits are two-dimensional fuzzy sets, which are neither necessarily convex nor connected. Therefore, solution schemes are challenged to provide not too conservative bounds and, however, determine the total influence of the uncertainty. Furthermore, solution schemes have to handle the implicit formulation of the stability limits. This contribution is a formulation of fuzzified stability limits general solution scheme, which are presented in Sec. 4.2. In addition, a fuzzy sensitivity analysis is briefly introduced in Sec. 4.3, which is well suited for the computation of sensitivities of the stability behavior with respect to the uncertain parameters. In order to illustrate the applicability and to show that the results provide meaningful insight into the systems, several exemplary systems are considered in this work.

2. Examples of systems with time delay

One of the problems in machining that receives a lot of interest from the research community is a phenomenon labeled 'regenerative chatter'. Repeated cutting of the same surface, often modeled by introducing a delay term into the system equation, can lead to self-excited vibrations. Possibly, instability, resulting in high-amplitude vibrations, may be detrimental on both workpiece quality and equipment. Instability must therefore be avoided. So-called stability charts help the operator to choose the appropriate process parameters for finding a stable operation. Typically, stability charts are calculated using time-domain simulations or discretization techniques like the semi-discretization method (SDM), which will be presented in Section 3.

This section introduces three different models of time-delayed systems, which shall be used here for the demonstration of the stability analysis including uncertainties in the system parameters. Two of the models are motivated by machining systems, i.e. the well-known and simple turning operation and a quite complex helical milling operation, respectively, as well as the Mathieu equation, which exhibits a very complex stability characteristics and, therefore, is often used as a benchmark system for methods of stability analysis.

2.1. Model of a turning operation

Probably the simplest model of a turning operation consists of the mass-spring-damper system shown in Fig. 1 with the single degree of freedom $x(t)$. It is often used to describe the vibration in direction of the chip thickness
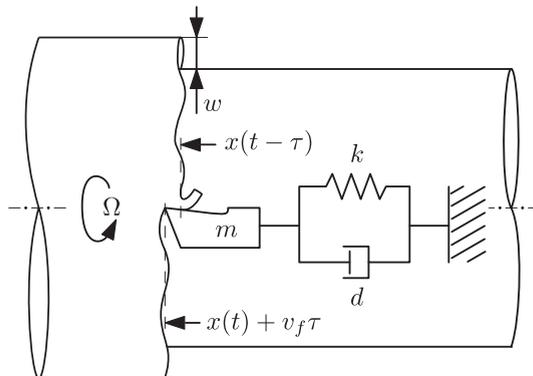

Fig. 1. Mechanical model of a turning operation.