Applied Thermal Engineering 125 (2017) 1157–1163

Contents lists available at ScienceDirect

Applied Thermal Engineering

journal homepage: www.elsevier.com/locate/apthermeng

Research Paper

Robust stability of thermal control systems with uncertain parameters: The graphical analysis examples

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HIGHLIGHTS

- A method for robust stability analysis of thermal control systems is presented.
- The graphical method is applicable for various systems with parametric uncertainty.
- The technique is extremely universal and relatively easy to use.
- Even very complicated systems (with non-existing analytical tools) can be analyzed.
- The robust stability is investigated with the necessary and sufficient condition.

ARTICLE INFO

Article history:
Received 20 February 2017
Revised 9 July 2017
Accepted 13 July 2017
Available online 15 July 2017

Keywords:
Robust stability analysis
Thermal systems
Parametric uncertainty
Time delay
Fractional order systems

ABSTRACT

This paper is intended to present the investigation of robust stability for integer order or fractional order feedback control loops affected by parametric uncertainty and time-delay(s) with special emphasis on the thermal control systems. The applied graphical method is based on the numerical calculations of the value sets and the zero exclusion condition. Three robust stability examples inspired by control of the real-world thermal processes are used for demonstration of the technique applicability. Namely, the work deals with the analysis of a shell-and-tube heat exchanger which was identified as the (integer order) time-delay model with parametric uncertainty, a heat transfer process modeled as the fractional order time-delay plant with parametric uncertainty, and a heating–cooling system with a heat exchanger described by the anisochronic model with internal delays and parametric uncertainty.

1. Introduction

Thermal systems arise in many engineering fields such as manufacturing, power generation, automotive and aerospace engineering, air conditioning, etc. [1] and so their efficient and low-cost control represent an important task. Beyond a doubt, stability is the most critical property of not only thermal) control systems. Thus, the investigation of the stability is widely studied discipline, which is going to be nontrivial especially if the control systems are affected by phenomena such as nonlinear behavior [2,3], time delays [4–7] or if they are described by means of fractional order derivatives and integrals [8–12]. Naturally, these factors can be often mixed and interconnected [13–15]. Moreover, the mathematical models of the controlled systems almost never exactly match their real-world behavior due to many reasons, e.g. linearization in an operating point, imperfections in modeling neglect of nonlinearities, "slow" time-variations, fast dynamic effects, etc.), changes in physical parameters or other perturbations. Respecting these influences in the mathematical description of the systems can lead to the use of the uncertain models and application of robust control [16–18]. Such uncertain model can be defined in more ways of which the parametric [19–22] and unstructured [23–26] approaches belong among the most commonly used ones. Anyway, if the control system is requested to remain stable for all possible members of the family defined via the uncertain model, one speaks about the robust stability.

The great interest of many researchers in robust stability analysis for systems with parametric uncertainty was boosted especially by the famous Kharitonov Theorem [27], or actually after its "rediscovery" for a wider automatic control community e.g. in [28]. In spite of the fact that Kharitonov Theorem represents the real milestone in the parametric robust control, its application is limited for the interval polynomials with mutually independent

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http://dx.doi.org/10.1016/j.applthermaleng.2017.07.089
1359-4311/

uncertain coefficients. However, the coefficients are often mutually dependent in practical systems and thus the tools for robust stability analysis of such systems were developed. The Edge Theorem, the 32 Edge Theorem (or similar Generalized Kharitonov Theorem), 16 Plant Theorem or Mapping Theorem belong among the best-known ones [16,17,21]. Their selection depends mainly on the level of dependency among the coefficients or other specific conditions. The research attention was paid also to the robust stability of time-delay systems [29–32] which can lead to the analysis of families of quasi-polynomials. Furthermore, the fractional order systems with parametric uncertainty have come under the spotlight recently [33–48]. Compared with the other methods, the graphical approach utilized in this paper, which is based on the value set concept and the zero exclusion condition [16,19], is relatively easy to use and it is particularly very universal, i.e. it can be applied from the simplest up to the very complicated uncertainty structures, including the quasi-polynomial families and fractional order cases. Moreover, the robust stability results are obtained without any conservatism (with the necessary and sufficient condition). On the other hand, a long computational time for a high number of uncertain parameters can be considered as the weakness of the technique.

As it has been already adumbrated, the thermal systems, as a complex class of plants and processes, are generally burden with all the mentioned negative effects, which makes their control complicated. There are various techniques for overcoming these obstacles in thermal systems control available. They include e.g. robust control [49,50], (robust) model predictive control [51], adaptive control [52], artificial neural networks, fuzzy control [53] and many others. Obviously, these methods are often interconnected as well. As the application of fractional calculus has become extremely popular in various engineering areas [11] recently, the thermal systems are no exception [54–57].

This paper presents the robust stability analysis for closed control loops with a fixed integer or fractional order controller (or two feedback controllers) and integer or fractional order controlled plant with parametric uncertainty and time-delay(s) with the special accent on the thermal control systems. The utilized graphical technique is based on the numerical calculations of the value sets and the application of the zero exclusion condition [16,19]. In order to show the usability of the method, three robust stability examples inspired by control of the real-world thermal processes are elaborated. Specifically, the paper contains the robust stability analysis of the feedback control loops with:

- A shell-and-tube heat exchanger which was identified as the (integer order) time-delay model with parametric uncertainty [50],
- A heat transfer process modeled as the fractional order time-delay plant with parametric uncertainty [56], and
- A heating–cooling system with a heat exchanger described by the anisochronic model with internal delays and parametric uncertainty [58].

To the best of authors' knowledge, the graphical analysis of robust stability for a loop with an anisochronic model with internal delays and uncertain parameters, presented in the last example, was not previously published to date.

This brief paper is organized as follows. In Section 2, the theoretical foundations of a graphical approach to robust stability analysis for integer order or fractional order systems with parametric uncertainty are presented. The key Section 3 then shows three graphical analysis examples for the thermal control systems with, successively, a shell-and-tube heat exchanger [50], a heat transfer process [56], and a heating–cooling system with a heat exchanger [58]. Finally, Section 4 offers some conclusion remarks.

2. A graphical approach to robust stability analysis under parametric uncertainty

The robust stability of the family of closed-loop thermal systems will be investigated through the robust stability of the family of its closed-loop characteristic polynomials (or to be more precise, retarded quasi-polynomials within this paper).

The family of continuous-time (fractional order or integer order) (quasi-)polynomials is [16]:

$$P = \{p(s,q) : q \in Q\} \tag{1}$$

where q is the vector of uncertainty, $p(s,q)$ is a (quasi-)polynomial with parametric uncertainty, and Q is the uncertainty bounding set. Most commonly, Q is assumed as a multidimensional box, which means that individual components of vector q are bounded by intervals. The family of (quasi-)polynomials (1) is robustly stable if and only if $p(s,q)$ is stable for all $q \in Q$.

Even for the "ordinary" integer order families of polynomials, the robust stability analysis can represent a nontrivial task. The critical factor for the decision on a convenient tool for investigation can be seen in the complexity of the coefficient functions in the polynomial $p(s,q)$. Generally, the more complicated relations among coefficients of the polynomial means the more complicated robust stability analysis. A classification of the uncertainty structures with growing generality (and complexity) and some typical tools for their robust stability testing can be found e.g. in [16,19,20,59].

The situation is even more complicated for the fractional order systems. However, several approaches have been already developed and the number is still growing [33–48].

The examples of thermal systems in this paper lead to various types of (integer order or fractional order) families of retarded quasi-polynomials. Some relevant methods are also available in the literature – e.g. [29–32].

Nonetheless, one graphical method seems to be unique from the viewpoint of its universality. It is based on the combination of the value set concept and the zero exclusion condition [16]. It can be applied to a wide range of uncertainty structures, from the simplest to very complicated ones, which suffer from the lack of suitable techniques. Furthermore, it is applicable also for various regions of stability (so-called robust D-stability). More details on parametric uncertainty and robust stability analysis and also examples of the typical value sets for the integer order systems can be found in [16] and subsequently e.g. in [19,20]. The works [33,34,38,39,43,44] extended the idea of the value set concept also to fractional order uncertain polynomials and the papers [59,48] showed their use for the specific class of families of fractional order uncertain quasi-polynomials.

According to the definition, the value set for the family of (quasi-)polynomials (1) at the frequency $\omega \in \mathbb{R}$ is [16]:

$$p(j\omega, Q) = \{p(j\omega, q) : q \in Q\} \tag{2}$$

which means that $p(j\omega, Q)$ is the image of Q under $p(j\omega, \cdot)$. In practice, the value sets can be constructed by substituting s for $j\omega$, fixing ω and letting the vector of uncertain parameters q range over the set Q.

The zero exclusion condition for the (Hurwitz) stability of the family of continuous-time (quasi-)polynomials (1) can be formulated [16]: Assume invariant degree of (quasi-)polynomials in the family, pathwise connected uncertainty bounding set Q, continuous coefficient functions, and at least one stable member $p(s, q^0)$. Then the family P is robustly stable if and only if the origin of the complex plane (zero point) is excluded from the value set $p(j\omega, Q)$ at all frequencies $\omega \geqslant 0$, i.e. P is robustly stable if and only if: