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Stability verification and timing contract synthesis for linear impulsive systems using reachability analysis☆

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ABSTRACT

This paper deals with stability analysis for a class of linear impulsive systems subject to a timing contract specifying bounds on the time between two consecutive impulses. We consider the problem of stability verification, which consists in proving stability for a particular timing contract, and the problem of timing contract synthesis, which consists in synthesizing a set of timing contracts that guarantee the stability of the linear impulsive system. Our approach is based on a reformulation using parameterized difference inclusions. We derive theoretical necessary and sufficient conditions for stability based on the propagation of a set by the system dynamics. For linear impulsive systems, when using approximate reachability analysis, this allows us to state a sufficient condition for stability and to design a stability verification algorithm. We then propose an approach to timing contract synthesis, which exploits the monotonicity of stability with respect to timing contract parameters to design an algorithm based on adaptive sampling of the parameter space. Several examples are provided, which allow us to compare our algorithm with several existing techniques, and show the effectiveness of our approach.

1. Introduction

Impulsive dynamical systems form a class of hybrid systems, which model processes that evolve continuously and undergo instantaneous changes at discrete time instants. Applications of impulsive dynamical systems include sampled-data control systems [1], networked control systems [2] or multi-agent systems [3]. In this paper, we deal with stability analysis of linear impulsive systems subject to a timing contract specifying bounds on the time between two consecutive impulses. We consider two problems of interest. The first problem under study is stability verification, which consists in proving stability of a linear impulsive system for a particular timing contract. We then consider the timing contract synthesis problem, which consists in synthesizing a set of timing contracts that guarantee stability of the system.

We use a reformulation of the linear impulsive systems in the general framework of difference inclusions. Then, for a fairly large class of difference inclusions, we establish necessary and sufficient conditions for stability. These conditions are based on the successive images of a set under the dynamics of the difference inclusion, and generalize some previous conditions on the stability of discrete-time switched systems [4,5]. For linear impulsive systems, these conditions allow

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Table 1
Some of the existing approaches for stability analysis of linear impulsive systems with description of the modeling and computational approaches.

Modeling	Computation	References
Difference inclusions	LMI	[12–14]
	SOS	[1,15]
	Set invariance	[11]
Time delay systems	LMI	[16–18]
Hybrid systems	LMI	[12,19]

us to design a stability verification algorithm using reachability analysis. Then, we take advantage of previous work [6], which provides an efficient and accurate algorithmic scheme to compute the reachable sets for linear impulsive systems. We then propose an approach to timing contract synthesis, which exploits the monotonicity of stability with respect to timing contract parameters to design an algorithm based on adaptive sampling of the parameter space, borrowing ideas from techniques for approximating the Pareto front of a monotone multi-criteria optimization problem [7,8].

Some results presented in this paper appeared in preliminary form in [9] for stability verification and in [10] for parameter synthesis. This paper widens their applicability by considering a more general formulation using difference inclusions, and provides technical details that were omitted in the previous works due to space limitation. Also, a new characterization of stability based on convexification of the difference inclusion is established (Theorem 7), which provides new insights regarding the conservatism of the proposed approach.

The paper is organized as follows. In Section 2, the problems under study are formulated and the relation between linear impulsive systems and difference inclusions is formally established. Section 3 addresses the stability verification problem. We provide new theoretical necessary and sufficient conditions for stability of a class of difference inclusions, which allow us to perform stability verification for linear impulsive systems using reachability analysis. Section 4 addresses the timing contract synthesis problem by combining the stability verification algorithm and adaptive sampling of the parameter space. In Section 5, examples are used to compare our technique with existing ones and to show the effectiveness of our approach.

Related work. Several approaches have been developed in the literature for stability analysis of linear impulsive systems. A non-exhaustive list is given in Table 1. From the modeling perspective, the problem can be tackled using difference inclusions, time-delay systems or hybrid systems. On the computational side, most of the approaches are based on semi-definite programming using either Linear Matrix Inequalities (LMI) or Sum Of Squares (SOS) formulations. This makes a clear distinction with our approach which relies on reachability analysis. [11] is more closely related to our approach since it relies on the computation, using backward reachability analysis, of sets which are contracting between two successive impulses. In comparison, our approach uses forward reachability analysis and we compute sets that may need several impulses before contracting. In Section 5, we will provide comparisons on numerical examples between our approach and those that are listed in Table 1.

Notations. Let $\mathbb{R}$, $\mathbb{R}_0^+$, $\mathbb{R}^+$, $\mathbb{N}$, $\mathbb{N}^+$ denote the sets of reals, nonnegative reals, positive reals, nonnegative integers and positive integers, respectively. For $I \subseteq \mathbb{R}_0^+$, let $\mathbb{N}_I = \mathbb{N} \cap I$. Let $\|\cdot\|$ be a norm on $\mathbb{R}^n$, and let $\mathcal{B}$ denote the associated unit ball. Given a real matrix $A \in \mathbb{R}^{n\times n}$, $\|A\|$ is the norm of A induced by the norm $\|\cdot\|$. Given $\mathcal{S} \subseteq \mathbb{R}^n$ and a real matrix $A \in \mathbb{R}^{n\times n}$, the set $A\mathcal{S} = \{x \in \mathbb{R}^n : (\exists y \in \mathcal{S} : x = Ay)\}$; for $a \in \mathbb{R}$, $a\mathcal{S} = (aI_n)\mathcal{S}$ where I_n is the $n \times n$ identity matrix. The convex hull of $\mathcal{S}$ is denoted by $\mathrm{conv}(\mathcal{S})$. We denote the set of all subsets of $\mathbb{R}^n$ by $2^{\mathbb{R}^n}$. We denote by $\mathcal{B}_0(\mathbb{R}^n)$ the set of bounded subsets of $\mathbb{R}^n$ containing 0 in their interior. For any $\mathcal{S} \in \mathcal{B}_0(\mathbb{R}^n)$, there exist $\underline{c}, \overline{c} \in \mathbb{R}^+$ such that $\underline{c}\mathcal{B} \subseteq \mathcal{S} \subseteq \overline{c}\mathcal{B}$. For $p, p' \in \mathbb{R}^d$, $p \leq p'$ if and only if $p_i \leq p'_i$, $i = 1, \ldots, d$.

2. Problem formulation

This paper mainly deals with stability analysis and timing contract synthesis for linear impulsive systems. We use a general formulation based on difference inclusions and later show how linear impulsive systems can be embedded in this framework.

2.1. Difference inclusions

We consider discrete-time dynamical systems modeled by the following difference inclusion:

$$z_{k+1} \in \Phi(\{z_k\}), \quad k \in \mathbb{N} \tag{1}$$

where $z_k \in \mathbb{R}^n$ is the state of the system, and $\Phi : 2^{\mathbb{R}^n} \to 2^{\mathbb{R}^n}$ is a set-valued map. Stability for systems of the form (1) is considered in the following sense:

Definition 1. System (1) is globally exponentially stable (GES) if there exists $(C, \varepsilon) \in \mathbb{R}^+ \times (0, 1)$ such that for all trajectories $(z_k)_{k\in\mathbb{N}}$ of (1), we have

$$\|z_k\| \leq C\varepsilon^k \|z_0\|, \quad \forall k \in \mathbb{N}. \tag{2}$$