Mechanical Systems and Signal Processing 96 (2017) 291–302

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

From homotopy perturbation technique to reduced order model for multiparametric modal analysis of large finite element models

F. Massa [a,*], I. Turpin [b], T. Tison [a]

[a] University of Valenciennes, LAMIH UMR CNRS/UVHC 8201, F-59313 Valenciennes, France
[b] University of Valenciennes, LAMAV EA 4015, F-59313 Valenciennes, France

ARTICLE INFO

Article history:
Received 15 February 2017
Received in revised form 12 April 2017
Accepted 19 April 2017
Available online 26 April 2017

Keywords:
Vibration
Projection
Homotopy
Perturbation
Reduced order model

ABSTRACT

The paper focuses on the definition of a reduced order model for linear modal analysis. The aim is to supply a suitable mathematical alternative tool compatible for multiparametric analysis of large finite element model considering numerous variable parameters, numerous mode shapes and significant levels of variation. The initial full eigenvalue problem is so replaced by a reduced one considering an efficient projection basis. To build it, we propose to combine homotopy transformation and perturbation technique for each parameter direction to define a reduced order model compatible with the design space. Finally, a complete finite element application highlights the capabilities of the proposal in terms of precision and computational time.

1. Introduction

To propose more and more efficient designs, multiparametric analyses, such as non-deterministic analyses [1], robust optimization [2,3], sensitivity analysis [4] are nowadays unavoidable. During the design phase, it is useful to investigate the behavior of mechanical solutions as a function of modification of material or geometric characteristics. The evolution of mass and stiffness are especially essential in vibration domain to optimize the structural resonance. To numerically study this problem, an associated linear eigenvalue problem is solved for example with an efficient Lanczos solver. However, when numerous samples depending on several variable parameters and numerous targeted eigensolutions are required, the computational time can be prohibitive and not compatible with a design phase. Thus, the proposition of a suitable mathematical alternative tool compatible with multiparametric analyses is an uptodate challenge for the scientific and industrial communities.

In the literature, many general numerical methods have already been proposed as alternative, such as Kriging method [4], Radial Basis Function [5], Proper Orthogonal Decomposition [6], in several application domains. The efficiency is clearly dependent on the number of snapshots, used to capture the evolution of the studied solutions. When numerous variable parameters and studied eigensolutions are investigated, the number of snapshots can be superior to several hundred to achieve an acceptable precision. Next, modal stability method [7,8] has been proposed to calculate modified eigenvalues with reduced computational time. The main drawback is relative to the quality of approximation because eigenvectors

* Corresponding author.
E-mail address: Franck.Massa@univ-valenciennes.fr (F. Massa).

http://dx.doi.org/10.1016/j.ymssp.2017.04.025

are assumed to be constant between nominal and perturbed configurations. Component Mode Synthesis [9] has been used as alternative technique too to reduce the computational time for the calculation of perturbed eigensolutions. Recently, a reanalysis technique [10] combining condensation and specific orthogonalization with the Rayleigh-Ritz analysis has been suggested in the case of simultaneous modifications. Moreover, many reanalysis techniques [11], successively based on combined approximations [12–14], perturbations [14,15] and series development [16,17], Padé approximants [18,19] and Taylor series expansion [20] have been developed too. The range of validity of these methods is globally linked to the level of variations introduced on input parameters. Finally, projection techniques [20–23] can achieve interesting approximation but the updating step of projection matrix for each new perturbation does not always allow to significantly reduce the computational time.

Thus, this paper is focused on the definition of a Reduced Order Model (ROM), which allows to approximate both eigenvalues and eigenvectors with a high level of precision and reduced computational time. The key idea is to decompose independently, for each variable parameters, the modified eigenvalue problem into a set of linear problems via a homotopy technique, to identify the associated high order eigenvectors as a function of input matrix perturbations and finally to integrate, in a projection basis, all the non collinear vectors representative of all variable directions.

Section 2 recalls the equations associated to the full linear modal problem and allows to define the reduced problem. Section 3 describes the strategy to calculate the high order eigenvectors according to homotopy transformation and perturbation technique. Sections 4 and 5 are respectively dedicated to the description of the reanalysis by projection techniques and the definition of the unique projection basis considering the aggregation of high order eigenvectors. Next, Section 6 concerns the numerical application. The capabilities of the proposed method are tested as a function of multiple parameters, such as the number of variable parameters and studied mode shapes, the order of truncation and the size of finite element models. Finally, Section 7 offers our conclusions.

2. Reduced eigenvalue problem

The modal analysis of a mechanical structure, discretized by the finite element method, is represented by an eigenvalue problem expressed as follows:

$$\mathbf{K}\mathbf{\Phi}_i = \mathbf{M}\mathbf{\Phi}_i\lambda_i \quad i = 1 \ldots n_{mod} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, $\mathbf{\Phi}_i$ and λ_i represent respectively the ith eigenvalue and eigenvector of the structure and n_{mod} the number of studied eigensolutions. The size of the finite element matrices $\mathbf{M}$ and $\mathbf{K}$ is $[n_{dof} \times n_{dof}]$, where n_{dof} is the number of degrees of freedom used to model the studied structure.

Each eigenvector $\mathbf{\Phi}_i$ is here calculated through mass normalization:

$$\mathbf{\Phi}_i^T\mathbf{M}\mathbf{\Phi}_i = 1 \tag{2}$$

Considering now an ad hoc rectangular projection basis $\mathbf{T}$, whose the size is $[n_{dof} \times n_{col}]$, the eigenvalue problem Eq. (1) can be reduced by considering a decomposition of the eigenvector $\mathbf{\Phi}_i$:

$$\mathbf{\Phi}_i = \mathbf{T}\mathbf{q}_i \tag{3}$$

where $\mathbf{q}_i$ is a reduced coordinate vector.

By introducing Eq. (3) in Eq. (1) and pre-multiplying by $\mathbf{\Phi}_i^T$, we obtain the classical reduced eigenvalue problem:

$$\hat{\mathbf{K}}\mathbf{q}_i = \hat{\mathbf{M}}\mathbf{q}_i\lambda_i \tag{4}$$

with $\hat{\mathbf{M}} = \mathbf{T}^T\mathbf{M}\mathbf{T}$ and $\hat{\mathbf{K}} = \mathbf{T}^T\mathbf{K}\mathbf{T}$. The size of matrices $\hat{\mathbf{M}}$ and $\hat{\mathbf{K}}$ is so $[n_{col} \times n_{col}]$.

Classically, the unknown data $\lambda_i, \mathbf{q}_i$ and $\mathbf{\Phi}_i$ are calculated for example with Lanczos algorithm.

The discussion, proposed in the following sections, is dedicated to the building of the projection basis $\mathbf{T}$.

3. High order eigensolutions calculation

This section is dedicated to the calculation of high order eigensolutions, which will be integrating in the projection matrix of the reduced order model.

Let us consider a modified eigenvalue problem defined by Eqs. (5) and (6). The exponent $^{(m)}$ indicates the modified contributions.

$$\mathbf{K}^{(m)}\mathbf{\Phi}_i^{(m)} = \mathbf{M}^{(m)}\mathbf{\Phi}_i^{(m)}\lambda_i^{(m)} \tag{5}$$

$$\mathbf{\Phi}_i^{(m)T}\mathbf{M}^{(m)}\mathbf{\Phi}_i^{(m)} = 1 \tag{6}$$

The modified matrices $\mathbf{M}^{(m)}$ and $\mathbf{K}^{(m)}$ can be decomposed as a function of nominal matrices $\mathbf{M}^{(0)}$ and $\mathbf{K}^{(0)}$ and associated perturbed matrices $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$. The nominal data are generally associated with the center of the studied design space.