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Fast analysis of multi-static scattering problems with compressive sensing technique

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ABSTRACT

In this paper, the compressive sensing (CS) is introduced to the electromagnetic multi-static scattering problems for the purpose of reducing computation runtimes. During the simulations, the CS paradigm is worked as an incident angle reduction tool. The scattered fields generated by all excitations in one scattering angle are treated as the signal of interest, which is under sampled directly based on the CS theory. And then the measurement of the scattered fields is converted to the measurement of the induced currents and the excitations by the Huygens principle and the relationships between the induced currents and the excitations. By solving the measured matrix equation with the fast multipole method (FMM), one can get the measurement of the scattered fields. Finally, the orthogonal matching pursuit (OMP) is utilized to reconstruct the original scattered fields by solving an optimal ℓ_1 norm equation. Numerical simulations demonstrate that, if the interpolation is not taken into account, the introduction of CS can save significantly while maintaining sufficient accuracy.

1. Introduction

To sample a signal in traditional data acquisition system, the Shannon–Nyquist sampling theorem must be followed. In another word, to avoid losing information when capturing a signal, the sampling rate must be at least twice the maximum frequency in the signal. With the increasing demand for information, the bandwidth of the signal is wider and wider. Thus, the sampling rate is becoming faster and faster. But in some applications, increasing the sampling rate is very expensive. Also the ever increasing amounts of data make it very difficult to transmission and storage. Therefore it comes so naturally that the sampled data must firstly be compressed to reduce transmission and storage costs. Consequently, the sample—then—compress framework causes a great waste of resources.

Compressive sensing (CS) is a new sampling paradigm that goes against the traditional sample-then-compress framework. CS was first proposed by Donoho, Candès and Tao in 2006 [1–4]. One of the most attractive properties of the CS theory is that: CS promises to overcome the sampling rate limitation enforced by Shannon–Nyquist sampling theorem and reconstruct signals from much fewer measurements than the nominal number of data points. As a consequence, CS has emerged as one of the most active areas in imaging [5,6], photography [7], audio/video capture [8], etc. Also, several electromagnetic problems are found to be suitably recast for an effective solution within the CS framework. For example, antenna arrays analysis and synthesis [9], inverse scattering [10], ground penetrating radar [11], etc. A broad review of CS in electromagnetics has been put forward by Massa et al. in [12,13].

In this paper, the electromagnetic multi-static scattering problem is interested, which is an important branch of electromagnetics. Generally, to perform such a multi-static scattering problem with the fast multipole method (FMM) when plane wave excitation is considered, one typically needs to firstly constitute and solve a matrix equation to derive the associated unknown induced currents for each incident angle. The computed induced currents are then being employed to compute the scattered fields at all scattering angles. As a result, the amount of computations is directly related to the number of incident angles. And unfortunately, the number of incident angles that must be considered in solving multi-static scattering problems is typically very large. This makes the solving of multi-static scattering problems a computationally challenging task. Therefore, how to reduce the incident angle sampling rate while maintaining sufficient accuracy is becoming more and more important. And this motivated the introduction of CS theory to scattering problems.

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There also exist several studies in the literature that utilizing the CS theory to reduce the number of computations required for solving scattering problems. For example, in [14–17] the CS theory is applied to speed up the matrix solving procedure of method of moments (MoM) in solving bistatic scattering problems. Unfortunately, the proceeded problems in the aforementioned studies have only one incident angle, which not belong to the scope of the wide angle scattering problems. In [18], Wu and his team proposed a CS-MoM method, which introduces the ideas from CS to the MoM, to efficiently solve two dimensional monostatic scattering problems. Based on Wu's work, Chai et al. [19,20] introduced the nonuniform rational B-spline (NURBS) surfaces and the Rao–Wilton–Glisson (RWG) basis functions into the CS-MoM to solve the three dimensional monostatic scattering problems. In [21], a CS based multilevel fast multipole algorithm (MLFMA) method is proposed, which introduces a new type of excitations, to further reduce the amount of data need for monostatic scattering. In [22], Cao attempts to use the excitation matrix as the sparse transform matrix in the CS-based MoM method. Chen proposed a CS-CBFM method in [23]. In CS-CBFM, the characteristic basis function method (CBFM) is utilized to reduce the matrix size and make the matrix equation easier to solve, while the CS is introduced to decrease the number of matrix equations that need to be solved for in performing a monostatic scattering analyzing. The application of CS to the iterative physical optics (IPO) is also demonstrated in [24] for the purpose of reducing simulating runtimes. In [25], the hybrid method that conjugating CS technique, MoM and the adaptive cross approximation (ACA) for efficiently target characterization is proposed and validated. All of the aforementioned methods are proposed for monostatic scattering problems. Although these methods can be expanded to multi-static scattering problems with minor modifications, they have a common disadvantage. That is the induced currents on one patch over all incident angles are set as the proceeded signal. Therefore the number of signals is equal to the number of unknowns in the methods mentioned above. And the number of unknowns is typically very large in scattering problems. Thus the above mentioned methods are all inefficient in analyzing multi-static scattering problems. In this paper, a new method named CS-FMM is proposed to improve the computational efficiency in solving the multi-static scattering problems. In CS-FMM, the FMM and the CS are utilized as the computing kernel and the incident angle reduction tool, respectively. It should also be pointed out that, the scattered far fields over all incident angles at a fixed scattering angle are treated as the proceeded signal in CS-FMM, and the number of the signals is equal to the number of scattering angles. It easy to be found that, the signal-form and the signal-number of the CS-FMM are both different with those of the aforementioned methods.

Carin et al. [26] have exploited an in situ CS algorithm, which conjugate the CS theory with the MLFMA, for fast computation of multi-static scattering problems. But the CS-FMM method proposed in this paper has its own advantages. First, the scattered far fields over all incident angles at a fix scattering angle are treated as the proceeded signal in CS-FMM. While the signal represents the Fourier transform of the induced currents in Carin's work. Second, a complex propagation medium and a window function need to be introduced to exploit the multipath of the medium and simplify the introduction of CS theory in [26]. But all these are not needed in our method. Hence, the CS-FMM proposed in this paper is much easier to understand and use. Third, the scattered fields derived by Carin's method include the effects of the induced currents introduced by interaction with the heterogeneous medium. But in CS-FMM, there is no such part of influence. Fourth, both the incident angles and the scattering angles are sparsely measured in Carin's method. While CS-FMM proposed in this paper only need to sparsely sample the incident angles. Fifth, discrete cosine transform (DCT) is chosen as the sparse basis function in [26]. But the fast Fourier transformation (FFT) is used in CS-FMM.

Another way to accelerate the existing algorithms is reducing the number of iterations in solving each matrix equations. For example, Okada et al. proposed an accelerating method of discrete dipole approximation (DDA) to solve matrix equations efficiently in [27]. In which the interpolation of the surrounding incident angle results is utilized as the initial guess of this incident angle. It is worth noting that the total number of incident angles (or matrix equations) is not changed in Okada's method. While CS-FMM concentrates on reducing of the number of incident angles (the number of matrix equations to be solved for). The number of iterations in CS-FMM is the same with that in FMM.

T-matrix [28,29] is another class of methods that solving scattering problems. There exist two differences between CS-FMM and T-matrix. First, T-matrix is based on spherical harmonics, while the sparse basis in CS-FMM can be set freely as DCT, FFT, DWT, etc. Second, the incident and scattered fields are both expanded into spherical vector wave functions in T-matrix. But the CS-FMM only needs to expand the scattered fields.

The remainder of this paper is organized as follows. In Section 2, a brief introduction of CS and FMM is provided first. Then a detailed procedure of how CS-FMM method worked is demonstrated. A comparison of the computational complexity of traditional FMM and the CS-FMM is also presented in this section. In Section 3, the CS-FMM is utilized to calculate the far zone scattered fields from several conducting bodies (cylinder, sphere, plate and ship). The computational results and the runtimes of the CS-FMM are compared with those of the traditional FMM to illustrate the validity and efficiency of the new method. Finally, concluding remarks are addressed in Section 4.

2. Theoretical model

2.1. Compressive sensing theory

Compressive sensing [1–3] attempts to sample a compressible signal at a rate significantly below the Shannon–Nyquist rate and reconstruct the signal without too much perceptual loss. Thus one of the fundamental of CS theory is that the signal to be dealt with is compressible or sparse. Assume that the processed signal is a one dimensional complex signal with length N. Rewrite the signal in column vector form as $x \in \mathbb{C}^{n\times 1}$. By introducing a proper sparse transform basis $\Psi_{N\times N}$, x can be expressed as

$$x_{N\times 1} = \sum_{i=1}^{N} \Psi_i \tilde{y}_i = \Psi_{N\times N}\tilde{y}_{N\times 1} \tag{1}$$

in which y represents the weighting coefficients. By k sparse, we mean that y has a concise representation and has only k nonzero elements.

In this paper, the FFT matrix is selected to be the sparse transform basis, which has the following forms

$$\Psi_{N\times N} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{N-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & w^{N-1} & w^{2(N-1)} & \cdots & w^{(N-1)(N-1)} \end{bmatrix}_{N\times N} \tag{2}$$

where $w = \exp(-2\pi i/N)$.

The second precondition that makes CS sampling and recovering possible is incoherent [30], which means that the measurement matrix $\Phi_{m\times N}$ is incoherent with the sparse transform basis $\Psi_{N\times N}$. Where m is the number of measurements. In another word, the rows of Φ cannot be sparsely represented by the columns of Ψ.