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On Structural Behavior of a Funicular Concrete Polyhedral Frame Designed by 3D Graphic Statics

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ABSTRACT

This paper investigates the mechanical behavior of the first built funicular structure designed by 3D graphic statics (3DGS) using reciprocal polyhedral diagrams. Graphic statics methods are unique among other structural design techniques in providing an intuitive control over the form of the structure and its equilibrium of forces for designers. Since graphic statics does not include material properties in the form finding process, further numerical investigations to foresee the behavior of the system under loading scenarios other than the 3DGS design loads are unavoidable. This research reports the structural behavior of Hedracrete which is a prefab, concrete, polyhedral frame with both compression and tension members. A simplified (bar-node) and a detailed volumetric mesh model of the structure are numerically analyzed under eight different loading scenarios. Different combinations of linear and nonlinear material behavior for steel and concrete in addition to linear and quadratic element types are used in those analyses. The analyses are planned to initially verify the equilibrium results of the 3DGS model and to predict the maximum load-bearing capacity of the structure by studying the failure mechanism of the system.

1. Introduction

Geometric structural design methods, known as graphic statics (GS), are considered among the most powerful design techniques that have been researched and practiced by many researchers and structural designers since the early nineteenth century [3,9,12,13,16,18,21,22,26]. Graphical methods, either used as procedural techniques or implemented computationally, result in structural concepts that are exemplary of structural efficiency and expressive form.

What makes GS methods unique among other structural design techniques is the unprecedented and intuitive control that it provides for designers; in GS, the form of the structure and its equilibrium of forces are represented by two diagrams known as *form* and *force* diagrams. These diagrams are *reciprocal* [18]; i.e. geometrically and topologically dependent. Thus, a change in one diagram results in a change in the other. This unique property allows designers to explicitly control the form of the structure as well as the magnitude of its internal and external forces.

1.1. Graphic statics: a brief development history

GS methods fall into three main categories; 2D, 2.5D, and 3DGS methods; 2DGS methods that are based on 2D reciprocal diagrams were originally proposed by Rankine [23], formulated by Maxwell [18] and developed by Culmann [13], Cremona [12], and many others [28]. Although their resulting structural forms are limited to 2D concepts, 2DGS methods were used by many eminent engineers and designers such as Guastavino, Maillart, Eiffel, Nervi, Dieste and their built structures are highly commended for their minimal use of materials [8,14].

Thrust Network Analysis (TNA) [10] is an example of the 2.5GS method combining 2D polygonal reciprocal diagrams and force density method [25] to generate breathtaking, funicular free-form shells which are generated/represented as height fields [27].

There are multiple extensions of GS in three dimensions; the methods that are based on i) projective geometry; ii) reciprocal (non-planar) polygonal diagrams and iii) reciprocal polyhedral diagrams. The methods that are based on projective geometry were mainly developed by Föpl [15] and can be used to analyze determinate 3D truss systems, but the complexity of the projective drawings can make it quite counter-intuitive for designers.

The GS method based on reciprocal (non-planar) polygonal diagrams was proposed by Maxwell [18] and Cremona [12] and the use of these methods based on Combinatorial Equilibrium Modelling (CEM)

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Fig. 1. A photograph of the built structure in Sa'dabad Complex, Tehran, Iran.

has recently been suggested by Ohlbrock et al. [21].

3D Graphic Statics Using Polyhedral Reciprocal Diagrams is the third category and will be referred to 3DGS in this paper. This method has recently been developed based on a 150-year-old proposition in Philosophical Magazine by Rankine [23], Akbarzadeh [2], Akbarzadeh et al. [4], and McRobie et al. [20]. As shown in Fig. 2, in 3DGS, the equilibrium of a node v_i of a polyhedral frame and its connected members/applied force(s) $e_{i,j}$ is represented by a closed polyhedral cell c_i whose faces are perpendicular to the members/applied force(s) of the node. The magnitude of the internal force $\mathbf{f}_{i,j}$ in each member is equal to the area of the corresponding face $f_{i,j}$ in the polyhedral cell [4]. In the following paragraphs, the form finding, materialization, and construction of this structure will be briefly explained to provide a foundation for further investigation in this paper.

1.2. Hedracrete: funicular polyhedral concrete

Hedracrete is a prefabricated, concrete polyhedral frame and the first built structure designed by the use of 3D graphic statics based on reciprocal polyhedral diagrams [2]. The form of the structure is a funicular polyhedral frame comprised of both compression-only and tensile-only members with the total height of 3.33 m, spanning from three supports located 5.4 m apart from each other. The structure consists of 129 prefabricated parts, 45 joints, 54 compression and 30 tension members sitting on steel supports connected by steel rods (Figs. 1 and 3). All parts were constructed from Glass Fiber Reinforced Concrete (GFRC) except the steel supports and the connectors of the tensile members. The total weight of the structure is 5480.6 kg where the heaviest joint and member weigh 103.8 kg and 126.8 kg, respectively [7].

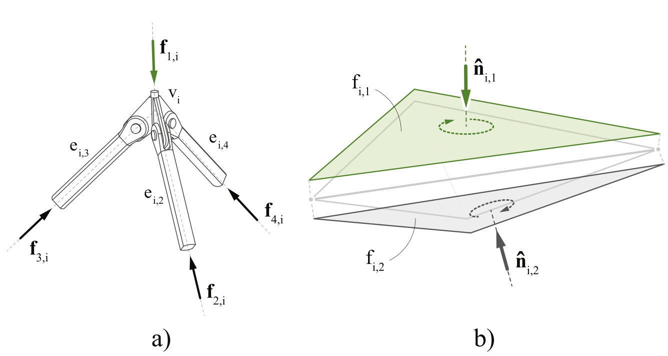

Fig. 2. (a) Node v_i of a spatial structure in equilibrium; and (b) the elements of the reciprocal cell c_i representing the equilibrium of node v_i, with directions of normals of the faces $f_{i,1}$.

1.3. Problem statement and objectives

Although 3DGS allows exploring static equilibrium of variety of non-conventional funicular solutions in three dimensions, its does not include material properties and self-weight of the members. Therefore, the mechanical behavior of the spatial funicular forms design by 3DGS must be evaluated using additional analytical models based on the assigned material properties and various loading cases other than the 3DGS design loads.

Hedracrete is the first application of 3DGS in constructing audacious spatial concrete structures. Thus, further investigations must be conducted to understand and predict the mechanical behavior of such systems. Consequently, this research sets a twofold objective:

- to validate the results of the applied 3DGS using numerical calculations;
- to investigate and predict the mechanical behavior of the built structure such as the type and magnitude of the internal stresses under its self-weight, ultimate load bearing capacity, and failure mechanism.

The following sections will expand on the structural form finding and fabrication process of Hedracrete followed by numerical analysis setup and the results for this funicular polyhedral frame.

2. Structural form finding

The development process of the form finding including the main idea and its development was explained thoroughly in Akbarzadeh et al. [7], and in this paper, we briefly explain the outcomes of the form finding process. The main objective in the form finding process was to find a *spatial* funicular form with both compression and tension members. There are some historic examples of expressive structures designed by 2DGS methods that are exemplary of the innovative use of material, construction technique and efficiency. However, their geometry is an extrusion of a 2D concept. Maillart's Chiasso Shed is an excellent example of such structures with both compression and tension members made out of concrete [29] (Fig. 4).

In 3DGS, the equilibrium of the external forces, which includes the applied and reaction forces, is controlled by the closeness of the Global Force Polyhedron (GFP) in the force diagram, and, the equilibrium of each node within the structure is represented by the closeness of a Nodal Force Polyhedron (NFP) [4,6]. As a rule of thumb, in a force diagram, if all NFPs are convex and contained within the volume of a GFP, there exists a form configuration with compression/tension-only members [3]. Whereas, if the volume of GFP does not contain all NFPs, there is a form configuration with both compressive and tensile forces [2,17].