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Global asymptotic stability analysis of two-time-scale competitive neural networks with time-varying delays

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A B S T R A C T

In this paper, the global asymptotic stability of two-time-scale competitive neural networks(CNNs) with multiple time-varying delays is investigated. By constructing a new ε-dependent Lyapunov functional, sufficient conditions for the global asymptotic stability of the concerned systems are established, and an optimization problem is formulated to get the best estimate of the ε-bound. Compared with the existing results, the proposed results are more general and less conservative in the sense of determining an upper bound for the time-scale parameter ε. Finally, three examples are given to illustrate the advantages of the obtained results.

1. Introduction

Competitive neural networks constitute an important class of neural networks and model the dynamics of cortical cognitive maps with a combined neural activity and weight dynamics [1]. Grossberg's shunting network proposed in [2] and Amari's model for primitive neuronal competition proposed in [3] are early competitive neural networks, where synaptic connections are considered to be fixed. In 1996, Meyer-Baese extended the earlier models to a special one whose synapses can be modified by external stimuli [1]. The model is represented by two differential equations and has two types of state variables with different time scales. One is the short-term memory(STM) describing the fast neural activity, and the other one is the long-term memory(LTM) describing the slow modifications of the synapses caused by external stimuli. The general equations describing the states for the ith neuron of an n-neuron CNNs with two time scales are as follows:

$$STM : \varepsilon \dot{u}_i(t) = -a_i u_i(t) + \sum_{j=1}^{n} \omega_{ij} g_j(u_j(t)) + b_i \sum_{k=1}^{n_p} m_{ik}(t) y_k, \quad (1)$$

$$LTM : \dot{m}_{ik}(t) = -m_{ik}(t) + y_k g_i(u_i(t)), \quad (2)$$

where $u_i(t) > 0$ represents the neuron current activity level, $m_{ik}(t)$ represents the synaptic efficiency, $g_j(u_j(t))$ is the output of neuron $u_j(t)$, $a_i > 0$ is the time constant of the neuron, ω_{ij} represents the connection weight between the i-th neuron and the j-th neuron, y_k is the constant external stimulus, b_i is the strength of the external stimulus and $\varepsilon > 0$ represents the fast time-scale associated with the STM state, $i, j = 1, \dots, n$, $k = 1, \dots, n_p$.

Neural networks model the dynamics of cortical cognitive maps accurately, and they can be widely applied to image processing, signal processing, optimization, pattern recognition and control theory, etc. [4]. Therefore, there have been a lot of research about the two-time-scale CNNs in the past years ranging from stability, synchronization and adaptive nonlinear systems identification, etc. (see [1,5,6] and the references therein).

Stability plays a key role in the design and application of neural networks [4]. And stability analysis of two-time-scale CNNs is essential and has been extensively studied [1,7–13]. Two-time-scale CNNs were first interpreted as nonlinear singular perturbed systems(SPSs) in [1] and a global stability analysis method was proposed by constructing a suitable quadratic-type Lyapunov function. Flow invariance theory was applied to study exponential stability of two-time-scale CNNs in [8]. Theoretical conditions ensuring global exponential stability of the network was established based on the nonsmooth analysis techniques in [9]. Lately, singular perturbation theory and vector Lyapunov function were employed to study the input-to-state stability(ISS) of two-time-scale CNNs in [10], and a method of estimating an upper bound for the singular perturbation parameter ε was proposed. However, since biological networks pose a certain degree of uncertainty or undergo many parametric perturbations, there arise some studies about the

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dynamical behaviors of the two-time-scale CNNs with fluctuating activation functions and synaptic weights [11,12] In [11], robust stability was studied and conditions were established based on the singular perturbation theory. A maximal upper bound for the fast time-scale associated with the neural activity state was derived. Stability conditions were given in [12] based on Gershgorin's Theory and a more realistic upper bound for ε is obtained. These existing models [1,7–12] were extended to deal with stochastic disturbance in [13], where the conditions ensuring the existence of the exponentially mean-square stability equilibria was established based on the theory of singularly perturbed stochastic systems.

In practice, time delays in artificial neural networks are very common due to the signal transmission lag and the finite switch speed of amplifiers in the circuit implementations, which may lead to oscillation, divergence, chaos, instability or other poor performance [14,15]. Thus, the dynamical behaviors analysis of two-time-scale CNNs with time delays has attracted a lot of attention in recent years [4,16–25] Single constant delay was first introduced into the two-time-scale CNNs in [16], and the global exponential stability was analyzed based on the nonsmooth technology in [17]. Time-varying delays was first introduced into the two-time-scale CNNs in [18], and the results were improved in [19] based on the LMI techniques. Some criteria for the CNNs with continuously distributed delays were presented by using theory of the topological degree and strict Lyapunov functional methods in [20]. Mixed delays containing time-varying delays and distributed delays was first introduced into the two-time-scale CNNs in [4], and the multi-stability in [4] and global exponential stability in [21] were investigated, respectively. In recent years, some researchers considered networks with time-varying delays and discontinuous activation functions [22–24]. The global stability was discussed by using the Leray–Schauder alternative theorem, LMI techniques and generalized Lyapunov function methods [22,23]. Global exponential stability of almost periodic solutions was studied based on nonsmooth analysis by constructing suitable Lyapunov functionals in [24]. These results were extended in [25] where mixed time-varying delays and discontinuous activation functions were considered at the same time. In addition, two-time-scale CNNs with multi-proportional delays were discussed and conditions for the exponential stability was obtained based on fixed point theorem in [26].

Two-time-scale CNNs were interpreted as SPSs and were decomposed as a lower-ordered reduced system and a boundary-layer system in [1]. A quadratic-type Lyapunov function was established as a weighted sum of individual Lyapunov functions for both subsystems, then an upper bound ε_0 was obtained. The ε-bound characterizes the robustness of system performance, and a larger ε-bound means better robustness [27]. Therefore, it is significant to consider and determine ε-bound. And many researchers concentrate on determining an upper bound ε_0 such that the concerned system is asymptotically stable for any ε less than or equal to the ε-bound [27–29]. However, among the existing literature about two-time-scale CNNs mentioned above, most of them do not emphasize the significance of ε-bound and ε is set as a constant (usually 1) for convenience, so that the obtained stability criteria hold only when ε is equal to the specific constant [4,9,22]. And the rest of them determine an upper bound for the time-scale parameter, so that the network is stable for any allowable value of ε [1,10,22] but time delays are not taken into consideration therein. The input-to-state stability of two-time-scale CNNs was studied by employing vector Lyapunov function in [10]. However, if considering time-varying delays, the analysis process would be very complicated. In [12], robust stability results were obtained assuming that the resulting parameter perturbations were only limited by their bounds. Therefore, to the best of our knowledge, there is no relative research considering ε-bound and multiple time-varying delays of the two-time-scale CNNs at the same time yet, which motivate this work.

In this paper, we will preserve the time-scale parameter ε in (1)–(2) and investigate the global stability of two-time-scale CNNs with multiple time-varying delays by considering the ε-bound. The main objective of this paper is to obtain an upper bound ε_0, such that the concerned network is globally asymptotically stable for any ε less than or equal to the ε-bound. And the main contributions of this paper can be summarized as follows. First, an ε-dependent Lyapunov functional is constructed and an LMI-based method is proposed to establish sufficient conditions for the global asymptotic stability of the concerned systems. An upper bound for the time-scale parameter ε is determined such that the system is globally asymptotically stable for any ε less than or equal to the ε-bound. And the method can be applied to solve the stability of two-time-scale CNNs without time delays. Second, an optimization problem is formulated to get the best estimate of the ε-bound. Compared with the existing results, our results are more general and less conservative.

The rest of this paper is organized as follows. In Section 2, problem formulation and preliminaries are provided. In Section 3, main results are proposed. Three examples are given in Section 4 to illustrate the advantages of the main results. And Section 5 concludes the paper.

Notations: The superscript T represents matrix transpose. For a column vector $x=[x_1,x_2,\dots,x_n]^T$, $\|x\|$ is the Euclidean vector norm, i.e., $\|x\|=(\sum_{i=1}^n x_i^2)^{1/2}$. If A is a symmetric matrix, $A>0$ means A is positive definite, and $A<0$ means A is negative definite. For any matrix A and two symmetric matrices P, Q, the symmetric term in a symmetric matrix is denoted by $*$, that is,

$$\begin{bmatrix} P & A \\ * & Q \end{bmatrix} = \begin{bmatrix} P & A \\ A^T & Q \end{bmatrix}.$$

2. Problem formulation and preliminaries

Let

$$\bar{S}_i(t)=\sum_{k=1}^{n_p} m_{ik}(t)y_k=\mathbf{y}^{\mathbf{T}}\mathbf{m}_i(t),$$

where

$$\mathbf{y}=(y_1,y_2,\dots,y_{np})^T,\ \|\mathbf{y}\|^2=y_1^2+y_2^2+\dots+y_{np}^2,$$
$$\mathbf{m}_i(t)=(m_{i1},m_{i2},\dots,m_{inp})^T.$$

Without loss of generality, assume the input stimuli $\mathbf{y}$ is normalized with unit magnitude $\|\mathbf{y}\|^2=1$, and take time-varying delays into account, then the state-space form of system (1) and (2) can be rewritten as follows:

$$\varepsilon\dot{u}_i(t)=-a_iu_i(t)+\sum_{j=1}^n\omega_{ij}g_j(u_j(t))+\sum_{j=1}^n\omega_{ij}^1g_j(u_j(t-\tau_{ij}(t)))+b_i\bar{S}_i(t), \tag{3}$$

$$\dot{\bar{S}}_i(t)=-\bar{S}_i(t)+g_i(u_i(t)), \tag{4}$$

where $u(t)=(u_1(t),u_2(t),\dots,u_n(t))^T$, ω_{ij}^1 is the interconnection weight of delayed feedback, $g_i(u_i(t))$ is the activation function, $\tau_{ij}(t)$ are bounded and differentiable time-varying delays satisfying $0<\tau_{ij}(t)\le\tau_{ij}^M$, $\bar{T}_i=diag(\tau_{i1}^M,\tau_{i2}^M,\dots,\tau_{in}^M)$, $\dot{\tau}_{ij}(t)\le\mu_{ij}<1$, $\eta_{ij}=1-\mu_{ij}>0$, $\circleddash_i=diag(\eta_{i1},\eta_{i2},\dots,\eta_{in})$, $i,j=1,2,\dots,n$, and the others are the same as those in (1) and (2).

Assumption 2.1. The activation function $g_i(u_i(t))$ is continuous and satisfies

$$0\le\frac{g_i(\eta)-g_i(\upsilon)}{\eta-\upsilon}\le\delta_i, \tag{5}$$