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Optimal online algorithms for the multi-objective time series search problem

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ABSTRACT

Tiedemann et al. (2015) [8] formulated multi-objective online problems and several measures of the competitive analysis, and showed best possible online algorithms for the multi-objective time series search problem with respect to those measures of the competitive analysis. In this paper, we present modified definitions of the competitive analysis for multi-objective online problems and propose a simple online algorithm Balanced Price Policy (BPP_k) for the multi-objective (k-objective) time series search problem. Under the modified framework, we show that the algorithm BPP_k is *best possible* with respect to any measure of the competitive analysis and we also derive best possible values of the competitive ratio for the multi-objective time series search problem with respect to several natural measures of the competitive analysis.

1. Introduction

Single-objective online optimization problems are fundamental in computing, communicating, and many other practical systems. To measure the efficiency of online algorithms for single-objective online optimization problems, Sleator and Tarjan [7] introduced a notion of competitive analysis. Since then extensive research has been made for diverse areas of single-objective online optimization problems, e.g., paging and caching (see [9] for a survey), metric task systems (see [5] for a survey), asset conversion problems (see [6] for a survey), buffer management of network switches (see [4] for a survey), etc. In practice, we have many online problems of multi-objective nature, however, general framework of competitive analysis and definition of competitive ratio for multi-objective online problems are not known. Tiedemann et al. [8] were the first to introduce a framework of multi-objective online problems as an online version of multi-objective optimization problems [2] and to formulate a notion of the competitive ratio for multi-objective online problems as the extension of the competitive ratio for single-objective online problems. To define the competitive ratio for multi-objective (k-objective) online problems, Tiedemann et al. [8] regarded multi-objective online problems as a family of (possibly dependent) single-objective online problems and applied a monotone function $f : \mathbf{R}^k \to \mathbf{R}$ to the family of the single-objective online problems. Let $\mathcal{A}$ be an algorithm for a multi-objective (k-objective) online problem. Then we regard the algorithm $\mathcal{A}$ as a family of

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algorithms $\mathcal{A}_i$ for the ith objective. For $c_1, \dots . c_k$, where c_i is the competitive ratio of the algorithm $\mathcal{A}_i$, we say that the algorithm $\mathcal{A}$ is $f(c_1, \dots, c_k)$-competitive with respect to a monotone function $f : \mathbf{R}^k \to \mathbf{R}$. In fact, Tiedemann et al. [8] defined the worst component competitive ratio, the arithmetic mean component competitive ratio, and the geometric mean component competitive ratio by monotone functions $f_1(c_1, \dots, c_k) = \max(c_1, \dots, c_k)$, $f_2(c_1, \dots, c_k) = (c_1 + \cdots + c_k)/k$, and $f_3(c_1, \dots, c_k) = (c_1 \times \cdots \times c_k)^{1/k}$, respectively.

1.1. *Previous work*

For the single-objective time series search problem (initially investigated by El-Yaniv et al. [3]), prices are revealed time by time and the algorithm tries to select a price as high as possible. Let $m > 0$ and $M > m$ be the minimum and maximum values of possible prices, respectively, and let $\phi = M/m$ be the *fluctuation ratio* of possible prices. For the case that m and M are known to online algorithms, El-Yaniv et al. [3] presented a (best possible) deterministic algorithm *reservation price policy* $\textsc{rpp}$, which is $\sqrt{\phi}$-competitive, and a randomized algorithm *exponential threshold* $\textsc{expo}$, which is $O(\log \phi)$-competitive.

Tiedemann et al. [8] defined the multi-objective time series search problem by a natural extension of the single-objective time series search problem. For the multi-objective (k-objective) time series search problem, a vector $\vec{p} = (p_1, \dots, p_k)$ of k (possibly dependent) prices is revealed time by time and the algorithm tries to select a good[2] price vector. For each $1 \le i \le k$, let $m_i > 0$ and $M_i \ge m_i$ be the minimum and maximum values of possible prices for the ith component, respectively, and assume that m_i and M_i are known to online algorithms. For each $1 \le i \le k$, let $\textsc{itv}_i = [m_i, M_i]$ be an interval of the prices for the ith component. Under the assumption that all of $\textsc{itv}_1 = [m_1, M_1], \dots, \textsc{itv}_k = [m_k, M_k]$ are *real* intervals, Tiedemann et al. [8] presented best possible online algorithms for the multi-objective time series search problem with respect to the monotone functions f_1, f_2, and f_3, i.e., a best possible online algorithm for the multi-objective (k-objective) time series search problem with respect to the monotone function f_1 [8, Theorems 1 and 2], a best possible online algorithm for the bi-objective time series search problem with respect to the monotone function f_2 [8, Theorems 3 and 4], and a best possible online algorithm for the bi-objective time series search problem with respect to the monotone function f_3 [8, §3.2].

1.2. *Our contribution*

As mentioned in Subsection 1.1, Tiedemann et al. [8] showed best possible online algorithms for the multi-objective time series search problem with respect to the monotone functions f_1, f_2 and f_3, however, the optimality of the algorithms is discussed separately and independently with respect to each of the monotone functions f_1, f_2 and f_3. In this paper, we present a simple online algorithm Balanced Price Policy $\textsc{bpp}_k$ for the multi-objective time series search problem with respect to any monotone function $f : \mathbf{R}^k \to \mathbf{R}$ and in Theorems 3.1 and 3.2, we show that the algorithm $\textsc{bpp}_k$ is *best possible* with respect to any monotone *continuous* function $f : \mathbf{R}^k \to \mathbf{R}$ under the assumption that all of $\textsc{itv}_1 = [m_1, M_1], \dots, \textsc{itv}_k = [m_k, M_k]$ are real intervals. In Theorem 4.1, we derive the best possible value of the competitive ratio for the bi-objective time series search problem with respect to the *existing* monotone function f_2, which disproves the result that the algorithm in [8, Algorithm 2] is best possible for the bi-objective time series search problem with respect to f_2. In Theorem 4.2, we derive the best possible value of the competitive ratio for the multi-objective time series search problem with respect to the *existing* monotone function f_3, which extends the result that the algorithm in [8, Algorithm 2] is best possible for the bi-objective time series search problem with respect to f_3. Finally in Theorem 4.3, we derive the best possible value of the competitive ratio for the multi-objective time series search problem with respect to a *new* monotone function $f_4(c_1, \dots, c_k) = \min(c_1, \dots, c_k)$.

2. Preliminaries

For any pair of integers $a \le b$, we use $[a, b]$ to denote a set $\{a, \dots, b\}$ and for any pair of vectors $\vec{x} = (x_1, \dots, x_k) \in \mathbf{R}^k$ and $\vec{y} = (y_1, \dots, y_k) \in \mathbf{R}^k$, we use $\vec{x} \preceq \vec{y}$ to denote a componentwise order, i.e., $x_i \le y_i$ for each $i \in [1, k]$. Note that $\preceq$ is a partial order on $\mathbf{R}^k$. We say that a function $f : \mathbf{R}^k \to \mathbf{R}$ is *monotone* if $f(\vec{x}) \le f(\vec{y})$ for any pair of vectors $\vec{x} \in \mathbf{R}^k$ and $\vec{y} \in \mathbf{R}^k$ such that $\vec{x} \preceq \vec{y}$. Let $\mathbf{R}_+$ be the set of positive reals.

2.1. *Multi-objective online problems*

From the framework of multi-objective optimization problems [2], Tiedemann et al. [8] formulated multi-objective *online* problems. In this subsection, we present multi-objective *maximization* problems (multi-objective minimization problems can be defined analogously).

For any integer $k \ge 1$, let $\mathcal{P}_k = (\mathcal{I}, \mathcal{X}, h)$ be a multi-objective (k-objective) maximization problem, where $\mathcal{I}$ is a set of inputs, $\mathcal{X}(I) \subseteq \mathbf{R}^k$ is a set of feasible solutions for each input $I \in \mathcal{I}$, and $h : \mathcal{I} \times \mathcal{X} \to \mathbf{R}_+^k$ is a function such that $h(I, \vec{x}) \in \mathbf{R}_+^k$ represents k objective values of a feasible solution $\vec{x} \in \mathcal{X}(I)$. For an input $I \in \mathcal{I}$, an algorithm $\textsc{alg}_k$ for $\mathcal{P}_k$ computes a feasible solution $\textsc{alg}_k[I] \in \mathcal{X}(I)$. For an input $I \in \mathcal{I}$ and a feasible solution $\textsc{alg}_k[I] \in \mathcal{X}(I)$, let $\textsc{alg}_k(I) = h(I, \textsc{alg}_k[I]) \in \mathbf{R}_+^k$

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[2] We use a "good" price vector to mean that it achieves a competitive ratio as low as possible with respect to the monotone function $f : \mathbf{R}^k \to \mathbf{R}$.

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