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Stochastic optimal control, international finance and debt

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This paper is dedicated to the memory of James Tobin (1918–2002), teacher and friend

Abstract

We use stochastic optimal control-dynamic programming (DP) to derive the optimal debt/net worth, consumption/net worth, current account/net worth, and endogenous growth rate in an economy – which could be a country, region or sector within a country. Unlike the literature that uses an intertemporal budget constraint or the Maximum Principle, the DP approach does not require perfect foresight or certainty equivalence. Our results are generalizations of the Merton model, and are explained graphically within a mean–variance context. Two examples are provided to illustrate the usefulness of our technique in predicting debt crises.
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1. Different approaches to intertemporal optimization in open economies

Several noteworthy debt crises have occurred in recent years. In the case of South East Asia in 1997, data on the credit rating of bonds issued in the first half of the 1990s suggest that investors in emerging market securities paid little attention to credit risk, or that they were comfortable with the high level of credit risk that they

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were incurring. [1] The compression of the interest rate yield spread prior to and the subsequent turmoil in emerging markets have raised doubts about the ability of investors to appropriately assess and price risk. In the US agricultural debt crisis case, the boom of the 1970s was stimulated by a substantial rise in crop prices resulting from inflation, the growth of export demand and availability of credit. A dramatic rise occurred in the ratio of debt/value added. In the fall of 1979, the Federal Reserve Board tightened its monetary policy to reduce inflation and interest rates soared. Moreover, the resulting appreciation of the US dollar exacerbated the decline in foreign demand for US agricultural exports. The net effect was that many farmers found that they were not able to service their debts. In the first half of the 1980s, bankruptcies, defaults and bank failures resulted. In each case, a benchmark is needed to evaluate to what extent a debt deviates from its optimal value.

In our model economies borrow to finance investment and growth as well as consumption. Debt to finance capital formation involves two risks. One is the return on domestic investment. The second is the variable interest rate on debt. A *benchmark* is provided to evaluate to what extent the debt deviates from its optimal value. The variables of interest discussed in this paper are the *optimal debt*, *current account*, *growth rate*, and *consumption*. Our technique is applicable to any open economy – which could be a country, region or sector within a country. In the concluding section, we provide two examples of the use of the technique. One concerns the debt crisis in emerging markets and the second, the US agricultural debt crisis.

Optimality conditions should satisfy several criteria: (a) they involve observable and measurable variables, (b) if followed, would maximize the value of sensible criteria and (c) do not produce very bad results if there is imperfect knowledge or errors of measurement. Several approaches have been used to derive optimality conditions in open economies. The dominant ones use either "an intertemporal budget constraint" (IBC) or the Maximum Principle of Pontryagin. It is recognized that these approaches are deficient [2] in satisfying criteria (a)–(c) above.

As a rule, economists have used the Maximum Principle of Pontryagin to derive optimal control laws. This is an "open loop" type of optimization method that yields an entire sequence of controls to be followed from initial conditions. Half of the initial conditions must be obtained from transversality conditions which imply the solution of differential equations. Given the likelihood of unpredictable disturbances, errors of measurement, formulation and implementation, the overall system will not be stable unless converted into a feedback form. This is to be expected since the optimal path to the desired target is unique. It is clearly advantageous in economics to derive policies in feedback form, where the next move depends upon the current state, since these types of policies are self-correcting and robust to perturbations. [3]

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[1] In the Asian crises, spreads hardly increased in the months prior to the floatation of the Bhat. The credit rating agencies and the market analysts all failed to signal the Asian crises in advance. They downgraded these economies only after the crises. See International Monetary Fund (1997, 1998, 1999a,b).

[2] See Gandolfo (2001, ch. 1 and 18), Hahn and Solow (1995), and Stein and Paladino (1997).

[3] This was the contribution of Infante and Stein (1973).