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Interfaces with Other Disciplines

Almost budget balanced mechanisms with scalar bids for allocation of a divisible good

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ABSTRACT

This paper is about allocation of an infinitely divisible good to several rational and strategic agents. The allocation is done by a social planner who has limited information because the agents' valuation functions are taken to be private information known only to the respective agents. We allow only a scalar signal, called a bid, from each agent to the social planner. Yang and Hajek [Yang, S., Hajek, B., 2007. "VCG-Kelly mechanisms for allocation of divisible goods: Adapting VCG mechanisms to one-dimensional signals", IEEE Journal on Selected Areas in Communications 25 (6), 1237–1243.] and Johari and Tsitsiklis [Johari, R., Tsitsiklis, J. N., 2009. "Efficiency of scalar-parameterized mechanisms", Operations Research 57 (4), 823–839.] proposed a scalar strategy Vickrey–Clarke–Groves (SSVCG) mechanism with efficient Nash equilibria. We consider a setting where the social planner desires minimal budget surplus. Example situations include fair sharing of Internet resources and auctioning of certain public goods where revenue maximization is not a consideration. Under the SSVCG framework, we propose a mechanism that is efficient and comes close to budget balance by returning much of the payments back to the agents in the form of rebates. We identify a design criterion for *almost budget balance*, impose feasibility and voluntary participation constraints, simplify the constraints, and arrive at a convex optimization problem to identify the parameters of the rebate functions. The convex optimization problem has a linear objective function and a continuum of linear constraints. We propose a solution method that involves a finite number of constraints, and identify the number of samples sufficient for a good approximation.

1. Introduction

This paper is about allocation of an infinitely divisible good to several strategic agents. The social planner who does this allocation has limited information in the sense that the agents' valuation functions are taken to be private information known only to the respective agents. We allow only a scalar signal from the agents to the social planner, which we call a bid. This is the only means by which agents can provide information about their valuation functions to the social planner. We are interested in an *efficient* mechanism: the allocation should maximize the sum of valuations of the agents. Under these constraints, we study mechanisms that come close to budget balance. Example situations described next, include fair sharing of Internet resources, disbursal of funds by a parent department, and auctioning of certain public goods, where revenue maximization is not a consideration.

Example 1. A communication channel with total capacity C is to be shared among several rational and strategic agents. This channel can be allocated via a randomized allocation rule, and is thus an infinitely divisible resource. If an agent gets a long term average throughput of a_i, the agent's valuation is $v_i(a_i)$, where $v_i : [0, C] \to \mathbb{R}_+$ is increasing, concave, and known only to the agent. Naturally, $\Sigma_i a_i \le C$. The agents wish to share the resources among themselves without money transferred to an external agent. Suppose that the agents agree to communicate with an external coordinator who attempts to maximize the sum of valuations. The signal space complexity to signal the valuation functions to the coordinator is prohibitive, particularly when the agents are geographically separated, because the functions can be arbitrary within the infinite-dimensional class of increasing concave functions. To model this communication constraint, we assume that the agents can send only a scalar signal. In this example, the coordinator is the social planner who desires efficient allocation without an interest in maximizing revenue. The scalar signals are viewed as bids.

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Example 2. A parent organization has to disburse available funds (assumed divisible) among several of its departments. Each department has a certain valuation function v_i for the allocation, is strategic, and the parent department desires to allocate efficiently while retaining only a minimal balance, if at all, based on limited information that the departments provide. Consider the extremely limited information setting of a scalar signal. The parent department is the social planner, the scalar signals are the bids, and the parent department desires an efficient distribution and no surplus.

The Vickrey–Clarke–Groves (VCG) mechanism (Clarke, 1971; Groves, 1973; Vickrey, 1961) achieves efficient allocation, but only when the signal space is sufficiently complex to describe entire valuation functions. In the VCG mechanism, the social planner requests agents to submit their valuation functions. The social planner then allocates to maximize the sum of the submitted valuation functions and determines the agents' payments.

Motivated by the communication network context but with nonstrategic agents, Kelly (1997) proposed a mechanism that involved only scalar bids. Under the Kelly mechanism, the social planner first collects scalar bids from the agents. Then the social planner allocates the good in proportion to the bids, and collects payments equal to the bids. The price per unit, or the market clearing price, is the sum of the bids divided by the quantity of the good. Every agent sees the same market clearing price. This distributed solution was shown to be efficient under certain conditions, but the agents should be price-taking or nonstrategic. If the agents are strategic, there is an efficiency loss of up to, but not more than, 25% (Johari & Tsitsiklis, 2004).

The VCG mechanism payments involve prices per unit good that can differ across the agents. This is not the case in the Kelly mechanism. In order to reduce the efficiency loss in strategic settings with scalar bids, Yang and Hajek (2007) and Johari and Tsitsiklis (2009) brought the feature of price differentiation across agents (a feature of the VCG mechanism) to the Kelly mechanism. The resulting mechanism, a scalar strategy VCG mechanism[1] (SSVCG), was shown to have efficient Nash equilibria.

All the above mechanisms typically result in a budget surplus (sum of payments from agents is positive). In this paper, our ideal is to achieve *budget balance*, or zero budget surplus. However, simultaneously achieving efficiency and budget balance in a strategy-proof mechanism is, in general, not possible (due to the Green–Laffont theorem (Green & Laffont, 1977); see footnote 6).

In the VCG setting, where there is no constraint on signaling, various *almost budget balance* notions and associated mechanisms were proposed. Almost budget balance is achieved by redistributing the payments among the agents in the form of rebates. Guo and Conitzer (2009) and Moulin (2009) studied rebate design in the case of discrete goods. Gujar and Narahari (2009, 2011) studied rebate design for the allocation of m heterogeneous discrete goods among n agents. Chorppath, Bhashyam, and Sundaresan (2011) studied rebate design in the divisible goods setting.

A big advantage with the VCG setting is that the social planner comes to know the true valuation functions. Voluntary participation of agents, i.e., agents being better off by participating in the mechanism, is easily verified. Furthermore, knowledge of the valuation functions could be exploited in defining a criterion for almost budget balance, as is done in Moulin (2009) and Chorppath et al. (2011). The extension of the almost budget balance notion to the SSVCG setting, however, is not straightforward. We cannot assume that the valuation functions are available because agents supply only a scalar bid. We thus relax our objective to that of achieving Nash equilibrium instead of achieving the DSIC (Dominant Strategy Incentive Compatibility) property.

In this paper, we consider the SSVCG setting that allows the agents to send only a scalar bid. We (1) propose a notion of *almost budget balance* appropriate for the SSVCG setting, and (2) design an optimal mechanism as per the proposed notion of almost budget balance.

Kakhbod and Teneketzis (2012) designed a mechanism to achieve an efficient Nash equilibrium with no budget surplus, but considered a setting where the agents signal a two-dimensional bid to the social planner. Moreover, their mechanism may not be feasible when the signals of the agents are not at Nash equilibrium. Sinha and Anastasopoulos (2013) modified this mechanism to have feasibility even under off-equilibrium situations, but required agents to signal a four-dimensional bid to achieve strong budget balance at equilibrium. We are not aware of any mechanism that achieves an efficient Nash equilibrium with strong budget balance using only scalar bids.

There are several design choices that we will make in arriving at a criterion for almost budget balance in the SSVCG setting. Considerations of tractability and significant reduction in surplus will guide our design decisions. For example, we restrict attention to the so-called linear rebates. This is mainly because it makes the optimization problem analytically tractable. An additional reason for the choice of linear rebates is that they are known to be optimal in the homogeneous discrete goods setting of Moulin (2009) and Guo and Conitzer (2009). The best justification however is the significant reduction in the surplus seen in our simulation results.

The coefficients of the linear rebate functions will be determined by a solution to a convex optimization problem. Specifically, we need to solve an *uncertain convex program* (UCP) (Calafiore & Campi, 2005) involving a linear objective function and a continuum of linear constraints. We propose a solution method that involves a finite number of constraints, and provide guarantees on the number of samples needed for a good approximation. We first prove that, under some sufficient conditions, the solutions of a general UCP and its corresponding relaxed UCP are close. We then prove that the specific linear rebate UCP satisfies these sufficient conditions.

The rest of this paper is organized as follows. In Section 2, we discuss the problem setting and the SSVCG mechanism. In Section 3, we discuss design choices for almost budget balance and rebate functions, our design decisions, and formulate an optimization problem. In Section 4, we make crucial reductions that ensure that our proposal can be implemented. The resulting optimization problem is a UCP. In Section 5, we study a general UCP and formulate a sufficient condition for an approximate solution via sampling of constraints. In Section 6, we apply the solution of Section 5 to the UCP for almost budget balance. In Section 7, we summarize our results, discuss alternative choices, and suggest possible extensions. Some simulation results demonstrate the usefulness of our approach.

2. The setting

2.1. SSVCG mechanism

A social planner needs to allocate a unit divisible resource among n intelligent, rational, and strategic agents. Agent i has a *valuation function* $v_i : [0, 1] \to \mathbb{R}_+$ privately known only to herself. The interpretation is that if $a_i \in [0, 1]$ is the fraction of the good allocated to agent i, her valuation is $v_i(a_i)$. The social planner's goal

[1] For some examples of mechanism design with restricted signaling, see Reichelstein and Reiter (1988) (minimal strategy space dimension for fully efficient Nash equilibria), (Semret, 1999) (two-dimensional bids for each resource), (Jain & Walrand, 2010) (two-dimensional bids on bundles of resources), (Blumrosen, Nisan, & Segal, 2007) (number of bits needed for signaling the bid). Our focus however is on the one-dimensional signaling.