Regularity of a general equilibrium in a model with infinite past and future

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A B S T R A C T

We develop easy-to-verify conditions to assure that a comparative statics exercise in a dynamic general equilibrium model is feasible, i.e., the implicit function theorem is applicable. Consider an equilibrium equation, \( \Upsilon(k, E) = k \) of a model where an equilibrium variable \((k)\) is a continuous bounded function of time, real line, and the policy parameter \((E)\) is a locally integrable function of time. The key conditions are time invariance of \( \Upsilon \) and the requirement that the Fourier transform of the derivative of \( \Upsilon \) with respect to \( k \) does not return unity. Further, in a general constant-returns-to-scale production and homogeneous life-time-utility overlapping generations model we show that the first condition is satisfied at a balanced growth equilibrium and the second condition is satisfied for “almost all” policies that give rise to such equilibria.

1. Introduction

Policy evaluation in a general equilibrium model with overlapping generations is perceived as a daunting task, and our objective is to provide a transparent way to make it more reliable. It is common, even in the case of a partial equilibrium analysis, to first look for equilibrium using numerical methods, then calibrate or estimate the parameters of the model to fit a given data set, and only then to perform policy “experiments”, i.e., to evaluate the reaction of equilibrium variables given the calibrated parameters to a “shock” in policy, see, for example, Adjemian et al. (2011) for the description of the algorithm. In this paper we work with policies that are not limited to one-time shocks, rather, we allow them to be functions of time and of individual characteristics. We offer the ground work for an analytical approach: as in the classical textbook case à la Debreu, we show that implicit function theorem can be applied to an equilibrium equation \( (\Upsilon(k, E) = k) \) for “almost any” distribution of endowments \((E)\) that yields a balanced growth equilibrium, in which case the equilibrium variables (e.g., \( k \)) can be viewed as smooth functions of endowments \((E)\), thought of as a transfer policy.¹ Thus, the equilibrium responses generated by such comparative statics are data-independent and can be directly calculated from the specification of the model.²

The analytical approach we offer can be viewed as complimentary to the numerical one. Establishing local uniqueness (regularity) of equilibria validates the numerical approach by assuring that the close-by equilibrium after the policy change is unique; moreover, analytical approximation for the reaction of equilibrium to the policy change provides a robustness check for the parallel numerical calculation.

We find that the key sufficient condition for regularity of equilibria is time-invariance, implying, in particular, that time in the model should be the whole real line. Its alternative, the half-line, or assuming existence of some “starting point” at zero, prevents one from properly modelling perfect foresight, cf. Burke (1990), and might be responsible for indeterminacy, cf. Kehoe and Levine (1985) and Demichelis and Polemarchakis (2007), and hence inability to predict the effects of a policy. With time-invariance, verifying conditions needed for the implicit function theorem amounts to checking differentiability of the equilibrium map \((\Upsilon)\)

1 See Mertens and Rubinchik (2017) for the overview of the existing applications of ift to equilibria in infinite economies and the reasons why they cannot be applied to the overlapping generations models.
2 Note that objective here is distinct from a comparison across steady states or balanced growth paths, as in d’Albis (2007); Mierau and Turnovsky (2014); the latter can be useful, for example, in comparing across countries, each following its own path. Our analysis can be used to predict changes that will occur in the wake of an announcement of a new policy in a given economy.
and calculating Fourier transform of the derivative with respect to the endogenous variable \(k\).

This last step, based on Wiener’s theorem, \(\text{Wiener (1932)}\), was also used in \(\text{Mertens and Rubinchik (2017)}\), where generic regularity and stability was demonstrated for a parametric version of an overlapping generations economy with Cobb–Douglas production. The genericity argument there was provided with respect to the parameters of the model.

The paper consists of two main parts. First we formulate the result for an arbitrary fixed point equation and then illustrate our approach using an overlapping generations model with a general smooth constant–returns–to–scale technology and assuming only time-separability and homogeneity of life–time consumption.\(^3\)

This rather minimal set of assumptions allows for balanced growth equilibria, as is well-known in the literature (\text{King et al., 2002}). This is a base–line equilibrium, to which the policy change is applied, which can then trigger a non–stationary equilibrium response. Characterization of the equilibrium is based on \(\text{Mertens and Rubinchik (2013)}\). We demonstrate how to verify our sufficient conditions for the implicit function theorem in this case. In addition, we show that comparative statics is feasible for “almost any” transfer policy, i.e., the set of policies that is open and everywhere dense in the corresponding Banach space of policies. Proofs missing in the text, detailed presentation of the model (that summarizes \(\text{Mertens and Rubinchik (2013)}\)) and an explicit calculation of the equilibrium response of capital to a change in endowments are in the appendix.

2. The first formulation of the problem

Let \(P, V\) be the space of parameters and variables of an economic model respectively, both are Banach spaces. In the example that follows the space of parameters (transfers) is the space of locally integrable functions and the equilibrium variable (capital path) is a continuous uniformly bounded function defined on the real line (time).

Assume that for a given model, equilibrium conditions can be reduced to a fixed–point equation in variable \(k \in V\), given parameters \(E \in P\),

\[
F(k, E) \overset{\text{def}}{=} Y(k, E) - k = 0
\]

(1)

The objective of the modeler is to assure that one can evaluate the equilibrium response of \(k\) to the change in the policy parameter \(E\) at some equilibrium \(k^0\) given baseline policy \(E^0\).

To illustrate the idea, let us view \(E\) as a function of \(k\). While the initial, base-line, equilibrium is stationary. There, \(E^0\) is constant with time, though it can still be a non–trivial function of individual age, thus describing some fixed system of transfers across individuals of different ages, for example as in a pay–as–you–go pension system. Then the change in the policy parameter, \(\delta E\), describes how the transfer system is altered across ages and over time. The resulting reaction of equilibrium variables, in particular capital path, is what we want to calculate.

For finite economies such calculations are done by appealing to an implicit function theorem (IFT), which requires differentiability of \(F\) and invertibility of the derivative \(\frac{\partial F}{\partial k}\) at the base–line point \((k^0, E^0)\). We follow the same route here. First, let us state the suitable version of the implicit function theorem (IFT). The differentiability below is in the sense of Fréchet.

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\(^3\) Such models are indispensable if one has to evaluate the effect of a policy that involves an intergenerational transfer, such as a pension reform for example, cf. e.g., \(\text{de la Croix and Michel (2002)}\) for the overview.
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