On the belief structures and reductions of multigranulation spaces with decisions

Anhui Tan a,b, *, Wei-Zhi Wu a,b, Yuzhi Tao c

a School of Mathematics, Physics and Information Science, Zhejiang Ocean University, Zhoushan 316022, China
b Key Laboratory of Oceanographic Big Data Mining and Application of Zhejiang Province, Zhoushan 316022, China
c Scientific Research Department, Zhejiang Ocean University, Zhoushan 316022, China

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A B S T R A C T

Multigranulation spaces provide a unified framework for representing multigranulation knowledge. In this paper, we aim to construct belief structures and characterize knowledge reduction in terms of evidence theory for the multigranulation spaces where decision attributes are considered. First, we introduce a type-1 belief structure for a multigranulation space, and prove that a pair of lower and upper pessimistic multigranulation approximations correspond to a pair of belief and plausibility functions under this belief structure. Second, we point out that no belief structure can be induced when using the optimistic multigranulation approximations in general. By adding a special sufficient condition, a second type of belief structure is further explored to study those that cannot be induced. Third, we develop several measurements by using the belief and plausibility functions, and characterize knowledge reduction of multigranulation spaces based on these measurements. In the end, a numerical algorithm for reducing redundant structures in multigranulation spaces is designed. An example is used to examine the efficiency of the proposed algorithm.

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1. Introduction

Granular computing (GrC) [56] has emerged as one of the fastest growing information processing paradigms in computational intelligence. In GrC, rough set theory and evidence theory are two basic mathematical techniques for uncertainty analysis [6,23,28,29,36]. Rough set theory was originated from Pawlak [23] and has been generalized into various models for meeting practical challenges, such as the variable precision rough set, decision-theoretic rough set (DTRS) and game-theoretic rough set (GTRS) [7,51,54]. It should be mentioned that DTRS vastly facilitated the development of a theory of three-way decision (TWD) [49,50]. In fact, the idea of TWD has already appeared in many intelligent decision-making disciplines [10,22,37]. However, there was not a unified formal description until that a theory of TWD was outlined by Yao [50]. Yao [53] also introduced a trisecting-and-acting framework of TWD, in which TWD is described as two separated tasks of trisecting and acting. With respect to trisecting [53], a universal set is divided into three regions. With respect acting [53], three strategies are used to process the three regions. In theory, the researches on TWD mainly focus on three aspects, i.e., background researches, theoretical framework researches and application researches [14,15]. For example, Hu et al. [12–14]...
introduced generalized definitions of decision measurement, decision condition and evaluation function and presented a framework of three-way decision spaces. Hu's axiomatic approach is important for the study and development of TWD theory. Azam and Yao [1–3] employed the GTRS mechanism to choose appropriate decision making criteria for determining the regions of acceptance, rejection and deferment in TWD. Yao and Azam [46,47] studied medical decision support systems by using GTRS-based TWD. Liu [20] provided a novel TWD model in incomplete information systems. Yu et al. [55] applied TWD methods for automatically determining the number of clusters in complex data.

The theoretical framework of evidence theory (also known as Dempster–Shafer theory) is a belief structure, which consists of a function mapping from the power set of the universe to the unit interval, called a basic belief assignment on the power set, and a family of subsets called focal elements with associated individual positive weights summing to one. Based on the belief structure, a pair of belief and plausibility functions on the universe are defined to measure the belief and plausibility degrees of uncertain knowledge. It is noted by many scholars [9,40,41,44,48] that, rough set theory and evidence theory have close connections. The pair of rough set approximations can be interpreted by the pair of belief and plausibility functions in evidence theory, and knowledge reduction with rough sets can be characterized by the belief and plausibility functions. For instance, Skowron [32–34] studied the relationships between rough set approximations and belief functions in the framework of Pawlak approximation spaces. Yao [51] interpreted knowledge structures in terms of evidence theory and rough set theory. Wu [38,39] investigated the rough set reductions of information systems by using belief and plausibility functions. Chen et al. [4,5] designed numerical reduction algorithms of covering information systems by introducing the evidence theory-based characteristics of covering rough sets.

It is well known that, all these results mentioned above are with reference to a single granulation knowledge. As we know, more and more data come from different sources in real life applications, which would generate multigranulation knowledge by adopting appropriate multigranulation techniques in the field of GrC [17,31,43,52]. Under this context, Qian et al. [24–26] developed the notions of optimistic and pessimistic multigranulation rough sets (MGRS) for uncertainty description in the presence of multigranulation knowledge. In the same sense, Khan and Banerjee [16] introduced a pair of strong and weak multigranulation approximations. It is pointed out by Yao and She [52] that although their studies are based on different angles, the proposed notions are mathematically equivalent. These two studies disclosed new directions of research for dealing with multigranulation knowledge. For the convenience of our study, we unitedly call the two approximations by optimistic and pessimistic multigranulation approximations by following Qian’s work. Up to now, many extended models of MGRS have been proposed, such as decision-theoretic MGRS, incomplete MGRS, covering MGRS, neighborhood-based MGRS and so on [21,42,45]. Furthermore, MGRSs have been shown to be useful in many applications. From a multigranulation viewpoint, Liang et al. [18] designed an efficient feature selection algorithm. She et al. [30] investigated the topological structures of MGRS from the perspective of multi-granularity. Yao and She [52] classified various rough set models in a multigranulation space. Lin et al. [19] and Tan et al. [35] introduced information fusion methods by combining MGRS and evidence theory in information systems. Qian et al. [27] introduced the idea of TWD into multigranulation structures and developed a three-way decision model for representing multigranulation knowledge.

It should be pointed out that most studies on multigranulation knowledge are concentrated on model generalizations, internal structures and real applications. Few works have been done on the measurements and knowledge reduction. On the other hand, the evidence theory-based measurements of a single granulation knowledge have been widely investigated, which are further employed to characterize the knowledge reduction and to develop substantial decision making algorithms. This fact motivates our idea to explore the measurements of multigranulation knowledge and characterize knowledge reduction of multigranulation spaces in terms of these measurements. In this paper, we first construct two types of belief structures for a given multigranulation space, based on which the two types of multigranulation rough set approximations are measured by the belief and plausibility functions. Then, several evidence theory-based measurements are proposed to characterize the significance of granular structures. In the end, we design an evidence theory-based algorithm for reducing redundant granular structures in multigranulation spaces.

The remainder of the paper is organized as follows. In Section 2, we review some related concepts of multigranulation rough sets and evidence theory. In Section 3, we study the belief structures of multigranulation spaces under pessimistic and optimistic multigranulation approximations, respectively. In Section 4, we investigate the reductions of granular structures in multigranulation spaces from viewpoint of evidence theory. In Section 5, we design numerical algorithms for reducing redundant granular structures in multigranulation spaces by using belief and plausibility functions. In Section 6, we conclude this paper.

2. Preliminaries

The basic notations of multigranulation spaces, TWD, multigranulation rough sets and evidence theory are briefly reviewed in this section.

2.1. Multigranulation spaces

An information system is a triple $S = (U, A, \{V_a | a \in A\})$, where $U$ is the universe, $A$ is a set of attributes such that $a : U \rightarrow V_a$ for any $a \in A$, i.e., $a(x) \in V_a \times U$, where $V_a$ is called the domain of attribute $a$. If $A$ is the union of two kinds of attributes, $A = C \cup D$, where $C$ is the so-called condition attribute set, $D$ is the so-called decision attribute set and $C \cap D = \emptyset$. Please cite this article in press as: A. Tan et al., On the belief structures and reductions of multigranulation spaces with decisions, Int. J. Approx. Reason. (2017), http://dx.doi.org/10.1016/j.ijar.2017.05.005
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