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An evidential analysis of Altman *Z*-score for financial predictions: Case study on solar energy companies

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ABSTRACT

Altman *Z*-score has been a well-accepted model of predicting survivals and failures of manufactures since 1968. However, short of an underpinning theory causes a wide gap between asking and responding sides, which still has no effective solution. This research proposes a rough set approach to inducing granular evidence and solving evidential coefficients of financial ratios for the distressed companies. Empirically, the proposed approach is applied to a financial database, Taiwan Economic Journal, to analyze the solar energy industry during 2009–2014. The result shows the inferential evidence successfully serves as a basis for financial analysis and discloses that the profit efficiency of the distressed companies in Taiwan's solar energy industry had been declining.

1. Introduction

The prediction model of discriminating survivals and failures proposed by Altman's Z-Score (AZS) has been playing a headship, achieving up eleven thousand citations in Google survey on April 5th 2016 and 75–90% reliability [1,2]. This model has highly credible applications to various domains including merger and divestment activity, asset pricing and market efficiency, capital structure determination, the pricing of credit risk, distressed securities, bond ratings and portfolios, etc. [3]. On the other hand, AZS has been questioned about discrimination [4], underpinning theory [5,2], over-modelling [6], generalizability [7], the relative importance of variables [7,3], etc. The opposite arguments make confusions to stakeholders such as bankers, investors, asset managers, rating agencies, and even the distressed companies themselves.

AZS is a paradigm of statistical prediction. However, AZS lacks evidence to clear up doubts. The exploitation on AZS inquires a high accuracy and finds the high accuracy is little associated with the other discrimination methodologies [3]. Short of underpinning theories and only discursive evidence available imposes difficulties in relieving doubts. Without theoretical evidence, the difficulties could arise from sampling, data instability, relative importance of variables, generalizability in modeling, accurate classification, and so on [6,8]. Since 1968 till now, a variety of improvements have been made on AZS such as linear discriminant analysis and *F*-value to disclose statistical properties of groups [9], principal component analysis to synthesize variables [10], logarithm to reduce outlier possibilities [11], stepwise analysis to rank the importance of variables [11], linear and quadratic analysis to validate classification [11], neural networks for classification [12], aggregated and weighted rates for pricing [13], etc. However, the evidence analysis has not been proposed for AZS so far [2]. A potential challenge of AZS lies in short of a sufficient condition containing evidence to support its inferential reliability. *We call this sufficient condition as the inferential evidence (IE) because it can be obtained from inference but unavailable beforehand*. In this paper, the inference is an induction approach.

Rough set theory (RST) is the one possessing rich knowledge about evidence. For instance, Pawlak claims RST has more substantial connection with evidence theory (ET) than fuzzy sets [14]. However, handling with vagueness and uncertainty makes these two theories different. ET uses belief functions as a main tool, while RST makes use of sets: lower and upper approximations [15]. Later the indiscernibility of approximation was improved by

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the preference orders of the dominance-based rough set approach (DRSA) to reduce the inconsistency between relations [16–18]. But the inconsistency has not been completely solved by DRSA due to uncertainty (noise in the approximations) [19,20]. Up to date, IE of RST is still vague when required to link objects' properties, decision information, and rational knowledge from inferences.

As known, the accurate prediction requires rigid analysis, the rigid analysis requires precise evidence; therefore the accurate prediction requires precise evidence as a sufficient condition. The precise evidence usually is not available beforehand. We are motivated to solve IE of RST to be the precise evidence by an induction approach developed from DRSA. This induction approach is named evidential induction model (EIM); naming with 'evidential' in this paper is derived from reliable evidence. The major outputs of EIM are the granular evidence (GE: a granule of RST composed of evidence) and the evidential coefficients of financial ratios (ECFR). The coefficient represents the contribution degree of individual financial ratios in AZS. Based on GE and ECFR, we will simulate AZS, classify companies, induce features, and analyze financial patterns for the distressed companies. This methodology is named evidential analysis (EA) for AZS. In a case study, EA will be applied to analyze Taiwan's solar energy industry during 2009–2014 to enhance understanding.

Our research goal will fulfill EA on AZS. The fulfillment of EA will firstly solve ECFR then generate GE. Further, the analysis will advance to simulate AZS, compare the classification features, disclose the financial patterns, and estimate the time-series trends for the distressed companies. The literatures about IE are presented in Section 2. A case study about Taiwan economic journal (TEJ) from 2009 to 2014 will illustrate all terms aforementioned. The context structure follows EA framework in Section 3, EA application in Section 4, EA discussion in Section 5, and finally concluding remarks. All operations in this research have been successfully implemented in programming.

2. The literatures about IE and innovation

The idea of IE originates from the granule, a basic atom of knowledge, of rough set theory (RST) [21]. The granules of RST are constructed with objects' properties (indiscernibility [21], similarity [22], relation [23], or decision classes [24]). This paper proposes GE as the core component of IE containing knowledge of decision information, objects' properties, and inferences from a vague set. The decision information is given by decision makers (DM). The rest two are presented below:

Objects' properties: The objects' properties of RST can be indiscernibility [21], similarity [22], preference [24], etc. On the other hand, RST uses sets to express the objects' properties. The sets composed of objects' properties can be relations [25], approximations (observable or unobservable) [25], classes (dominating or dominated) [24], etc. In general, the objects' properties based on attributes cannot clearly specify a vague set. Therefore, approximations are used for the vagueness estimation.

Approximations: The approximations are a pair of boundary sets, i.e. lower and upper [25]. For analyses, a vague set X can be identified within the boundary sets, i.e.

$$\underline{P}(X) \subseteq X \subseteq \overline{P}(X)$$

where P represents an inference function about the boundaries of X based on attributes, $\underline{P}(X)$ represents the lower boundary set, and $\overline{P}(X)$ represents the upper boundary set. P of the vague set has two types of inferences depending on whether X is a partition by attributes [25] or decision information [26,17,27]. This paper only discusses the latter type about the inferences between attributes and decision information.

Inferences: The inference of RST and DRSA focuses on the approximations. The approximation inferences handling X partitioned by decision information include the quality of approximation [17,27], the confidence on decision information by the coverage measure [28,29], and the confidence on implicational premise by the certainty measure [28,29]. Based on the inferential results, the analysts can realize DM's preference from objects' properties and population.

The approximation inferences of the traditional RST provide distributed knowledge but they are not enough to serve as evidence. In general, RST has three difficulties in handling uncertainties. The first is to make the approximation inferences into a unique expression which can help human to understand the distributed knowledge. The second is that granules might be vague, uncertain, or even unavailable without converging the approximation inferences. The third is the uncertainty problems from the previous difficulties influence each other. In short, the cause of uncertainty could arise from not only inference but evidence. For instance, the evidence theory (ET) can combine different sources to arrive at a degree of belief about a proposition. The generalized Bayesian inference in terms of conditioning data and multiplying models is developed for the belief combination [30]. It uses evidence for inference but its uncertainty still rises with the quantity of imprecise evidence [31]. When the evidence is not available beforehand, the uncertainty of inference becomes a huge problem. This is the reason why IE of RST has been hard to solve.

Facing the uncertainty, uniqueness requirement, and unavailability of granules, EA aims to disclose IE of RST. The construction of EA has two parts. The first proposes hypothetical approximations $P'_j(X) : \underline{P}'_j(X) \subseteq X \subseteq \overline{P'_j}(X)$ to logically initiate an inference model within attribute j, i.e. EIM. $P'_j(X)$ represent the priori approximations of EIM before making inference. EIM is illustrated in Figs. 1 and 2. The implementation details of EIM are presented in Section 3.2. The second is about a posterior inference by converging approximation inferences. EP is a posterior probability of EIM. ECFR and GE can be resolved from EP. The hypothesis and the inferential objects are described below. Their mathematical design is presented in Section 3.2.2–3.2.4.

2.1. Granular evidence, GE

GE is defined to comprise decision information, an object, and a unique expression of the approximation inferences, symbolized as e and formulated by Eq. (1).

$$e_{j,k} = 1 \text{ or } 0 \text{ where } e_{j,k} = 1 \rightarrow \times,\ e_{j,k} = 0 \rightarrow \circ \tag{1}$$

where 1 means distress by the approximation inferences, 0 means non-distress, j indexes a variable, k indexes an object, the symbol '×' expresses the distress of the company k with respect to variable j, and the symbol '○' expresses the non-distress for the company. GE is required to play a component of approximation. Its population and DM's preference together can enhance the effectiveness of decision making. Converging the approximation inferences into a uniqueness plays the technical key to generate GE, described next.

2.2. Hypothetical approximation, $P'_j(X)$

The hypothetical approximations are abbreviated as $P'_j(X)$ which take a vague set X that can be partitioned by an attribute's