Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems ••• (••••) •••–•••

FUZZY sets and systems

www.elsevier.com/locate/fss

The problem of collective identity in a fuzzy environment

José Carlos R. Alcantud [a,*], Rocío de Andrés Calle [b]

[a] *BORDA Research Unit and Multidisciplinary Institute of Enterprise (IME), University of Salamanca, Spain*
[b] *BORDA Research Unit, PRESAD Research Group and Multidisciplinary Institute of Enterprise (IME), University of Salamanca, Spain*

Received 20 April 2015; received in revised form 15 February 2016; accepted 17 February 2016

Abstract

We define the problem of group identification in a fuzzy environment. We concentrate on the case where the society is required to self-determine the belongingness of each member to a specific group, characterized by a single attribute. In general terms, this case consists of a collective identity issue that can be regarded as an aggregation problem of individual assessments within a group. Here we introduce the possibility that both the original assessments and the resulting output attach partial memberships to the collectivity, for each potential member. We propose relevant classes of rules, and some are axiomatically characterized. Our new approach provides a way to circumvent classical impossibility results like Kasher and Rubinstein's.

Keywords: Group identification; Fuzzy profile; Fuzzy collective identity function; Choquet fuzzy collective identity function; *WOWA*-fuzzy collective identity function

1. Introduction

The purpose of this paper is to extend the study of the group identification problem in Social Choice by allowing the natural possibility of partial membership degrees of the individuals to the groups. To be more precise in this first work we focus on a clear-cut statement called the problem of collective identity.

In this regard we adhere to Kasher [23], who introduces a notion of collective identity that remains basically unchallenged. It first attempts to answer the question of how to define a social group, albeit in a non-formalized sense. Kasher and Rubinstein [24] build on this philosophical position and state the problem from the point of view of Social Choice. To this purpose, they pose the question as an aggregation problem where the views of every member of the society about every member determine a well-defined collective identity of the group. In formal terms, the social outcome of their problem is a (crisp) subset of the society. The success of their contribution is that they show how algebraic aggregation theory (cf., Rubinstein and Fishburn [31]) intervenes to provide answers to such problem about aggregation of individual opinions. Among other achievements, they prove a celebrated impossibility theorem under

* Corresponding author.
E-mail addresses: jcr@usal.es (J.C.R. Alcantud), rocioac@usal.es (R. de Andrés Calle).

http://dx.doi.org/10.1016/j.fss.2016.02.011
0165-0114/

fairly mild conditions. Therefore although the use of aggregators (in the sense of [23] or [24]) and of aggregation operators (e.g., in the algebraic framework posed by [31] in economic theory) is not the same, the first relates to the second at the outset.

Clearly, an inspection of the relevant literature shows that the modeling of how individuals in a society are collectively viewed as belonging to a group is naturally linked to vague attributes, e.g., 'belonging to a newly formed nationality' (Dimitrov, Sung and Xu [16]), 'being Jewish' (Kasher [23]), 'being African-American' or belonging to any racial group (Miller, [28]), 'living in a rich neighborhood' (Dimitrov [14]), etc. Nevertheless the Social Choice literature has almost universally neglected this fact in the analysis of how social groups should be defined. By incorporating the subjectivity associated with general and vague attributes in the form of partial memberships we provide a richer environment where either the personal views of the individuals or the resulting collective identity or both are not necessarily dichotomic any more. In this framework we can better capture the subtleties of the problem, also because new families of aggregation operators can be employed to determine the social outcome (which can now be a fuzzy subset of the society).

Regarding this issue, besides the few traditional aggregation rules from the Social Choice literature we introduce the discrete Choquet integral, ordered weighted averaging operators (OWA), and weighted OWA operators (WOWA) as new tools that may determine the degree of memberships of the individuals to the collective identity. We utilize these aggregation functions due to their versatility and their well-known properties, that we proceed to briefly review.

The Choquet integral is the integral of a real function with respect to a fuzzy measure, by analogy with the Lebesgue integral which is defined with respect to an ordinary (i.e., additive) measure. Under the premise that the measure of the whole set is one, the discrete Choquet integral can be regarded as a mean or an averaging aggregation operator. In fact Marichal [26] presents the Choquet integral as an appropriate extension to the weighted arithmetic mean for the aggregation of criteria. Although it offers a large flexibility, somehow it keeps a linear form. Marichal proves how the Choquet integral can be characterized axiomatically by means of natural properties in [26, Section IV.B]. He offers an intuitive approach to it in [26, Section IV.A]. Several graphical interpretations of the Choquet integral are available from the literature, e.g., in Grabisch et al. [21] and Torra and Narukawa [37]. Earlier uses of Choquet integrals as aggregation functions in other contexts can be found e.g., in Calvo, Mayor and Mesiar [9, pp. 224–244], Grabisch [18], Grabisch et al. [19], Grabisch and Labreuche [20], and Marichal [27].

A particular instance of the discrete Choquet integral is the OWA operator (cf., Yager [38]). It is similar to a weighted mean, which synthesizes the values of the information sources according to their respective reliabilities, the difference being that in OWAs the values of the variables are previously ordered in a decreasing way. Therefore now the weights are not associated with concrete variables, and OWAs can diminish the importance of extreme values in order to increase the relevance of central ones. Calvo, Mayor and Mesiar [9], de Andrés Calle et al. [13], Llamazares [25] (in the context of Social Choice), and Yager and Kacprzyk [40] among others supply examples of applications of the OWA operator. From the point of view of the aggregation of opinions, it preserves anonymity while keeping the attractive of different weights.

An aggregation procedure that combines the advantages of using weights both to discriminate the sources of information and to calibrate the relevance of values in relation to their relative position is the WOWA operator (cf., Torra [34]). This procedure relies on an interpolation method (see also Torra [35]).

We can now be more explicit about the main target of this paper. We provide evidences that extending the analysis of the problem of collective identity by the introduction of partial memberships permits to better capture the nuances of preference modeling. Several frameworks apropos the fuzzy environment are presented and their properties are studied. We show that it is possible to preserve the nature of the investigations in Social Choice (e.g., the axiomatic treatment of the models) and to avoid restrictions imposed by the dichotomic viewpoint of the problem. We prove these benefits by characterizing the rules that evaluate each individual's membership to the collectivity by a discrete Choquet integral, and by producing a positive escape from Kasher and Rubinstein's impossibility theorem. Therefore our paper is in continuation of earlier successful developments on collective aggregation of fuzzy sets or fuzzy relations like e.g., Barrett [5], Barrett, Pattanaik and Salles [6], Dubois and Koning [17] and Montero [29].

The outline of our exposition is as follows. Section 2 explains the origin of the problem, its antecedents, and the precise framework in terms of crisp or categorical opinions that we investigate. In Section 3 we define our model for the fuzzy collective identity problem, allowing for various degrees of generalization. Our main tool of analysis, namely, *fuzzy collective identity functions* (FCIFs), is put forward and some direct examples are presented which extend the traditional collective identity functions. Then in Section 4 we propose a newly designed approach which

Please cite this article in press as: J.C.R. Alcantud, R. de Andrés Calle, The problem of collective identity in a fuzzy environment, Fuzzy Sets Syst. (2016), http://dx.doi.org/10.1016/j.fss.2016.02.011