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Divide and compromise

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HIGHLIGHTS

- Two symmetrized versions of the divide-and-choose mechanism are introduced.
- For a partnership dissolution between two agents, these mechanisms implement a balanced market outcome.
- Equilibrium outcomes do not depend on the identity of the first mover in the mechanism.

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ABSTRACT

We introduce two symmetrized versions of the popular divide-and-choose mechanism for the allocation of a collectively owned indivisible good between two agents when monetary compensation is available. Our proposals retain the simplicity of divide-and-choose and correct its ex-post asymmetry. When there is complete information, i.e., agents know each other well, both mechanisms implement in subgame perfect equilibria a unique allocation that would be obtained by a balanced market. The results hold for general continuous preferences that may not be quasi-linear.

1. Introduction

We consider the equitable allocation of a collectively owned object (indivisible good) when monetary compensation is available between two agents who know each other well, as in the dissolution of a 50%–50% owned family business.[1] Our main contribution is the introduction of two modified versions of the popular divide-and-choose mechanism. One mechanism resembles a natural sequential price negotiation in which both agents have the opportunity to make proposals and reach an agreement—see below for a precise description. The other mechanism is a two-step application of the divide-and-choose mechanism. Each subgame perfect equilibrium of our mechanisms results in an equitable compromise independently of the order in which proposals are made. Our proposals preserve the simplicity of the divide-and-choose mechanism without being biased towards the first mover.

The availability of equitable mechanisms is important in a market society. Economists have acknowledged the fundamental role of social trust for creating cooperation and have studied which factors are more relevant to determine the level of trust in a society. Ostrom (2000) points out that one of the key factors is the emergence of fair rules.[2] Productive activities are often conducted by groups of individuals who join their effort to achieve common goals. Thus, economic growth is indeed fostered by economic and social institutions that favor welfare enhancing exchanges, trades, and business agreements.

Our aim then is to identify equitable mechanisms. Our first step is to identify equitable allocations. In order to do so one can find an intuitively equitable institution and then select the optimal allocations that in ideal conditions the institution would produce. In our case, this is achieved by a market in which each agent, thought to

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1 Symmetric two-party partnerships are the modal form of business cooperation. For instance, Hauswald and Hege (2006) find that the majority of US joint ventures recorded by the Thomson Financial Securities Data in the period 1985–2000 are 50%–50% agreements.

2 "Fair rules of distribution help to build trusting relationships, since more individuals are willing to abide by these rules because they participated in their design and also because they meet shared concepts of fairness". (Ostrom, 2000, page. 150).

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be a price taker, owns half of the aggregate income. These allocations, which we refer to as equal-income market allocations, capture much of our desiderata of equity. They are efficient (Svensson, 1983). Moreover, since agents have the same income, they maximize in identical budget sets. Alternatively, in order to identify equitable allocations one can simply declare a desiderata of properties that an equitable allocation should have and then find the allocations that satisfy it. It turns out that requiring efficiency and that no agent should prefer the allotment of the other, i.e., the celebrated no-envy (Foley, 1967; Varian, 1974), exactly conduces to the set of equal-income market allocations (Svensson, 1983). With this solid foundation we concentrate on the implementation of equal-income market allocations.

Our second step is to account for agents' incentives. It is well known that it is impossible to implement equal-income market allocations in dominant strategies (Alkan et al., 1991; Tadenuma and Thomson, 1995a). In view of this impossibility, one can construct games whose Nash equilibrium outcomes are equal-income market allocations. An intuitive way to do this is by means of a so called α-auction: ask agents to bid for the object; then a highest bidder gets the object and transfers an α-convex-combination between the winner and the loser bid (Cramton et al., 1987; Brown and Velez, 2016).[3] Alternative simultaneous proposals abound. Unfortunately, it is well known that the performance of simultaneous move mechanisms is compromised by the presence of boundedly rational players (McKelvey and Palfrey, 1995). Indeed, the α-auctions perform poorly in an experimental environment (Brown and Velez, 2016). This leads us to consider fully sequential mechanisms. The most popular alternative here is the so called divide-and-choose mechanism (Crawford and Heller, 1979), which resembles the popular cake cutting procedure and implements in subgame perfect equilibria the "extremes" of the set of market outcomes.[4] Here, an agent chosen at random proposes the transfer that the agent who gets the object gives the other agent. The second agent decides either to get the object and make the proposed transfer, or to give up the object and take the transfer. In any subgame perfect equilibrium, the proposer takes advantage of her role and extracts all possible "equity surplus" from the other agent. This ex-post asymmetry turns out to be problematic. In laboratory experiments subgame perfect proposals are received with a retaliation strategy from the chooser, who can induce a big loss for the proposer at a low cost to him by just choosing the inefficient outcome (Guth et al., 1982; Brown and Velez, 2016). This welfare loss is significant (Brown and Velez, 2016).

We are, hence, interested in solving both limitations of simultaneous move mechanisms and the divide-and-choose mechanisms. We proceed in two steps. First, we identify a market outcome that, away from the extremes chosen by divide-and-choose, is a compromise that balances the interests of both agents within the set of equal-income market allocations. One can argue that at each equal-income market allocation each agent perceives a bias towards herself. This bias can be measured for, say agent i, by the maximal amount of money that one can add to the consumption of the other agent without causing agent i to prefer the other's allotment (Tadenuma and Thomson, 1995b). We select the equal-income market allocation at which the perceived biases of both agents are equal, which is essentially unique. We refer to it as the balanced market allocation.[5] Then, we construct two simple fully sequential mechanisms that implement in subgame perfect equilibria the balanced market allocation.[6]

Our first mechanism works as follows. An agent, say agent A, announces to be either the buyer or the seller and proposes a price (the outcome of the game is independent of the identity of the first mover). Suppose agent A announces to be the buyer and proposes price p_A. Agent B can either, steal A's proposal and buy at p_A—which ends the game, or renegotiate and propose a price p_B. If agent B renegotiates, agent A can then either steal B's proposal and sell at p_B—which ends the game, or compromise and buy at the average between p_A and p_B. If agent A announces to be the seller and proposes price p_A, the symmetric game unfolds. An interesting feature of this mechanism is that its subgame perfect equilibria exhibit an intuitive feature of situations in which agents compromise. In equilibrium agents make proposals that one can characterize as extreme. However, their extreme proposals balance each other and an equitable compromise, the balanced market outcome, is reached. The second mechanism we propose is a two-step application of the divide-and-choose mechanism (i.e., use the divide-and-choose principle to assign proposer role in the divide-and-choose mechanism).[7] Curiously, one may think that this mechanism leads to the middle point of the set of equal-income market allocations, but it actually leads to the balanced market allocation independently of who the first mover is.

Our implementation result is obtained in a domain of preferences that contains, but is not restricted to, quasi-linear preferences. As long as agents' preferences are increasing in money, there will be an essentially unique balanced market allocation, which is implemented in subgame perfect equilibria by our mechanisms. This level of generality, rarely found in implementation results, allows us to account for common phenomena as the complementarity of money and objects, or the natural asymmetry between making or receiving a money transfer under liquidity constraints (see Example 1).

2. Related literature

The equitable allocation of indivisible goods when monetary compensation is available has been the object of an extensive literature. Existence of equal-income market allocations has been established under very mild assumptions on preferences (Svensson, 1983; Maskin, 1987; Alkan et al., 1991; Velez, 2016a). Even though the set of equal-income market allocations generically has a continuum of allocations, none of the popular axioms of solidarity, monotonicity, and consistency has produced any focal selection from the set. Due to this indeterminacy several authors have proposed selections from this set based on intuitive criteria (e.g., Tadenuma and Thomson, 1995b; Aragones, 1995; Abdulkadiroğlu et al., 2004; Velez, 2011). Our balanced market allocation is indeed the allocation selected by the "equal-compensation solution" of Tadenuma and Thomson (1995b). Thus,

3 The online dispute resolution system http://www.fairoutcomes.com/ offers the intermediate price auction, i.e., $\alpha = \frac{1}{2}$, under the Fair Buy–Sell system.

4 The divide-and-choose mechanism has been a focal point in the fair cake division literature (Brams and Taylor, 1996) and has been adapted to multiple environments (Crawford and Heller, 1979; Crawford, 1980; Moulin, 1981; Thomson, 2005). Closely related to our results, Nicolò and Yu (2008) propose a mechanism that obtains envy-free allocations in the cake division problem. The mechanism is a multi-step sequential game form in which each agent at each step receives a morsel of the cake that is the intersection of what she asks for herself and what the other agent concedes.

5 Our selection has the property that for a fixed preference profile, the welfare of each agent is an increasing function of the aggregate consumption of money (Velez, 2016b).

6 Our approach is close in spirit to LiCalzi and Nicolò (2009) who identify a unique egalitarian-equivalent allocation for the land division problem and implement it in subgame perfect equilibrium.

7 The idea of auctioning the proposer role in the divide-and-choose mechanism was advanced first by Crawford (1979) for the allocation of a bundle of infinitely divisible goods.