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Arithmetic, working memory, and visuospatial imagery abilities in children with poor geometric learning

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A B S T R A C T

Many children fail in geometric learning, but factors underlying these failures have not been explored in detail. The present study addresses this issue by comparing fifth and sixth-grade children who had good or poor geometric learning, and were otherwise comparable on verbal intelligence, gender and age. Results showed that children with poor geometric learning have deficits in both arithmetic and geometric problem solving but they are more impaired in the latter. Results also showed that poor geometric learners have weaknesses in working memory, calculation, and visuospatial mental imagery. The results from logistic regressions pointed out that mental imagery skills and arithmetic problem solving ability had the highest discriminatory power in distinguishing between the two groups. Theoretical and practical implications of this research for designing interventions to help poor geometric learners are discussed.

1. Introduction

The complex set of acquisitions involved in learning geometry, including, for example, knowledge about spatial arrays and their measurement, are linked to students' future academic and professional success (Verstijnen, van Leeuwen, Goldschmidt, Hamel, & Hennessey, 1998). In fact, geometry represents one of the most important forms of mathematical knowledge, relevant in many aspects of everyday life (Cass, Cates, Smith, & Jackson, 2003) and important in fields including science, technology, engineering, and mathematics (Zhang, Ding, Stegall, & Mo, 2012). Nowadays, geometry is included in the majority of mathematical curricula in the world (OECD, 2010).

A relevant body of evidence on students who face specific difficulties in arithmetic, despite having average intelligence and sufficient achievement in other academic areas, has been collected (e.g., Passolunghi & Mammarella, 2010). Conversely, little evidence on students with specific difficulties in geometric learning is available (Mammarella, Giofrè, & Caviola, 2016). Consequently, the cognitive profile of students with difficulties in learning geometry has not been studied in depth. The goal of the present research is to provide insights on factors affecting difficulties in learning geometry.

Recent evidence proposes a distinction between intuitive geometry and geometric learning. Intuitive geometric concepts (e.g., Euclidean geometry) are shared by humans regardless of formal education (Dehaene, Izard, Pica, & Spelke, 2006; Spelke, Lee, & Izard, 2010). In contrast, the geometric learning explored in the present research, operationally defined as the ability to answer typical geometric questions and problems encountered in schools (Giofrè, Mammarella, & Cornoldi, 2014), involves concepts that are predominantly learnt through formal instruction. Geometric learning demands an explicit knowledge of principles and concepts (e.g., diagonals, parallel lines, and right angles) and of rules and their application in representing complex spatial relationships (e.g., imagining the result of the combination of two figures). Such learning also involves applying rules to specific requests (e.g., calculating the area or the perimeter of a figure). Due to this intrinsic complexity, school curricula in the early grades are focused on basic geometric knowledge (i.e., properties and rules that apply to plane figures such as circles, squares and triangles). Only later on, usually during the fifth and sixth grades, does the curriculum become more complex and structured, and this can create increasing difficulties for some students. It is worth noting, however, that a difficulty in learning geometry may be due not just to complexities involved in geometric learning. This kind of learning difficulty can also stem from a variety of factors that also seem to affect complex geometric learning, including calculation skills, working memory (WM), visuospatial mental imagery, and arithmetic problem solving ability. However, to what extent these aspects are associated with a failure in geometric learning has not been investigated in depth.

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A difficulty with calculation seems to be relevant because it impacts students' confidence as they cope with other types of mathematical situations (Aydın & Ubuz, 2010), including processes crucial for geometric learning. In particular, arithmetic is typically involved in many geometric situations requiring the use of measures and calculation (Mammarella et al., 2016). Also, a general problem solving ability is clearly connected to geometric learning, as it is associated with several distinct processes, such as comprehending the problem, building a representation of it, and planning and supervising the solution process (Mammarella et al., 2016; Passolunghi & Pazzaglia, 2004). In particular, arithmetic problem solving, involving not only calculation but also mathematical reasoning, may have a particularly strong impact on geometric learning. In addition, geometric tasks, due to their specific visuospatial features, may require specific abilities, which are not necessarily shared with arithmetic abilities (implied in calculation and arithmetic problem solving), such as spatial skills (e.g., Clements & Battista, 1992) and in particular visuospatial WM (Giofrè, Mammarella, Ronconi, & Cornoldi, 2013) and visuospatial mental imagery (Mammarella et al., 2016). As a result, arithmetic abilities may be necessary but not sufficient for children to master geometry.

In psychological literature, the role of WM has been widely acknowledged in arithmetic learning (e.g., DeStefano & LeFevre, 2004) but examined minimally in relation to geometry. However, WM seems to be involved in geometric learning not only because arithmetic and geometric problem solving share several WM resources (Passolunghi, Cornoldi, Liberto, Passolunghi, & De Liberto, 1999; Passolunghi & Mammarella, 2010; Zheng, Swanson, & Marcoulides, 2011), but also because geometric learning typically requires the temporary maintenance and treatment of both verbal and visuospatial information. This temporary maintenance can be seen, for example, in tasks such as representing geometric forms or memorizing specific geometric formulas (Giofrè et al., 2014). In fact, it has been shown that WM predicts success in school-related tasks that require the maintenance and processing of information, such as reading comprehension (e.g., Carretti, Borella, Cornoldi, & De Beni, 2009; García-Madruga et al., 2013), approximate mental addition (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; Mammarella et al., 2013), multi-digit operations (Heathcote, 1994), magnitude representation (e.g., Pelegrina, Capodieci, Carretti, & Cornoldi, 2014) and mathematical achievement (e.g., Bull, Espy, & Wiebe, 2008; Passolunghi, Mammarella, & Altoè, 2008). Because of this, it seems plausible to hypothesize that WM is similarly involved in learning geometry.

Working memory is a limited-capacity system that enables information to be temporarily stored and manipulated. In the classical dominant tripartite model of WM, the central executive is considered responsible for controlling resources and monitoring the processing of information across domains (Baddeley & Hitch, 1974). In contrast, the storage of information is mediated by two domain-specific slave systems: the phonological loop, which handles the temporary storage of verbal information, and the visuospatial sketchpad, which is specialized in retaining and manipulating visual and spatial information (Baddeley, 1996). A complementary approach distinguishes between many different types of WM processes based not only on the content of the information (visual, spatial and verbal), but also on the degree of cognitive control (Cornoldi & Vecchi, 2003). This distinction has been shown to be particularly relevant in the arithmetic domain, in which different WM components have varying involvement in arithmetic (Mammarella, Pazzaglia, & Cornoldi, 2008). Also, verbal, visuospatial and WM aspects may require different levels of cognitive control, and this distinction seems to be particularly relevant when considering geometric learning. As for visual and spatial tasks, spatial WM seem to require cognitive control to a lesser extent, while other visual WM tasks seem to require more attentional resources (Cornoldi & Vecchi, 2003).

Geometry deals with spatial information of two and three-dimensional patterns. According to recent reports, visuospatial WM may have a critical role both in arithmetic (Li & Geary, 2013; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013) and in geometric processes (Giofrè, Mammarella, Ronconi, & Cornoldi, 2013). Geometry involves processing of figures in space, and it seems plausible that, besides visuospatial WM, other visuospatial abilities affect geometric learning (Hannafin, Truxaw, Vermillion, & Liu, 2008). In particular, it has been argued that geometric learning can be sustained by visuospatial mental imagery (Weckbacher & Okamoto, 2014), which allows people to generate mental representations of geometric figures as they are verbally described and to manipulate, organize and compare elements across imagined figural patterns. In fact, visuospatial mental imagery is not only supported by visuospatial WM processes (Cornoldi & Vecchi, 2003), but also involves other skills related to the mental manipulation of forms (Andrade, 2002; Cornoldi, De Beni, & Mammarella, 2008) that may be crucial in geometric learning. Accordingly, a significant correlation between visuospatial mental imagery and geometry has been reported in high-school students, whereas the correlation between mental imagery and algebra was not statistically significant (Weckbacher & Okamoto, 2014).

The present study aimed to investigate which factors underlie the difficulties some children have in geometric learning. To reach this goal, and to identify both factors that cause difficulty in geometric learning and factors that support high geometric achievement, we adopted a good vs. poor ability design (also known as extreme group design). This approach is in fact very common for testing individual differences (Engle, 2010) and has been used extensively and successfully in several studies (e.g., Borella, Ludwig, Fagot, & De Ribaupierre, 2010; Fukuda & Vogel, 2011; Kane et al., 2007; Kane & Engle, 2002; Smeding, Darnon, & Van Yperen, 2015; Unsworth, Schrock, & Engle, 2004). Two groups of children - respectively with good and poor performance in a standardized geometry test but matched for age and verbal intelligence, and with no history of socio-cultural challenges, severe arithmetic difficulties, or clinical problems - took part in the study. Children were tested with a large set of tasks related to skills including WM, visuospatial mental imagery, calculation and arithmetic problem solving.

The separate consideration of geometric vs. arithmetic problem solving abilities has not received attention in the literature to date, also because, in other past research there has seldom been a differentiation between children with arithmetic difficulties only, children with geometric difficulties only, and children with difficulties in both areas (Mammarella et al., 2016). Therefore, for the present study, we developed arithmetic and geometric problems that were very similar, in terms of the solving procedures and the computation required, but crucially differed in their content (i.e., arithmetic or geometric). As the skills required for solving both geometric and arithmetic problems are partly overlapping, we hypothesized that children with poor geometric learning would struggle with both geometric and arithmetic problems. However, as the skills required for solving geometric problems also involve specific geometric abilities, we expected children with poor geometric learning to show greater impairment in the geometric problems compared to the arithmetic ones. If confirmed, this result would demonstrate that the difficulties that affect geometric problem solving are not entirely the same as those that affect comparable arithmetic problems.

To examine the role of calculation skills, WM, and visuospatial mental imagery in geometric learning, we compared children with good or poor geometric learning using a large set of tasks including a calculation battery, a WM memory battery, a visuospatial mental imagery test, and a problem solving battery (distinguishing between geometric and arithmetic problems). The arithmetic battery included simple and complex arithmetic calculations, and an approximate calculation task requiring children to decide which choice –between two numbers– better approximated the actual result of a series of calculations. The series of WM tasks assessed both verbal WM in its less controlled (forward digit span) and more controlled (backward digit span) components. We hypothesized that there would be statistically significant differences in almost all the domains, but that differences would be greater in mental imagery and problem solving. In fact, mathematical