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Two dimensional modeling of elastic wave propagation in solids containing cracks with rough surfaces and friction – Part I: Theoretical background

Vladislav Aleshin [a], Steven Delrue [b,*], Andrey Trifonov [a], Olivier Bou Matar [a], Koen Van Den Abeele [b]

[a] Joint International Laboratory LICS/LEMAC, Institute of Electronics, Microelectronics and Nanotechnologies, UMR CNRS 8520, Villeneuve d'Ascq, France
[b] Wave Propagation and Signal Processing Research Group, KU Leuven Kulak, Kortrijk, Belgium

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A B S T R A C T

Our study aims at the creation of a numerical toolbox that describes wave propagation in samples containing internal contacts (e.g. cracks, delaminations, debondings, imperfect intergranular joints) of known geometry with postulated contact interaction laws including friction. The code consists of two entities: the contact model and the solid mechanics module. Part I of the paper concerns the modeling of internal contacts (called cracks for brevity), while part II is related to the integration of the developed contact model into a solid mechanics module that allows the description of wave propagation processes. The contact model is used to produce normal and tangential load-displacement relationships, which in turn are used by the solid mechanics module as boundary conditions at the internal contacts. Due to friction, the tangential reaction curve is hysteretic and memory-dependent. In addition, it depends on the normal reaction curve. An essential feature of the proposed contact model is that it takes into account the roughness of the contact faces. On one hand, accounting for roughness makes the contact model more complicated since it gives rise to a partial slip regime when some parts on the contact area experience slip and some do not. On the other hand, as we will show, the concept of contact surfaces covered by asperities receding under load makes it possible to formulate a consistent contact model that provides nonlinear load-displacement relationships for any value of the drive displacements and their histories. This is a strong advantage, since this way, the displacement-driven model allows for a simple explicit procedure of data exchange with the solid mechanics module, while more traditional flat-surface contacts driven by loads generate a complex iterative procedure. More specifically, the proposed contact model is based on the previously developed method of memory diagrams that allows one to automatically obtain memory-dependent solutions to frictional contact problems in the particular case of partial slip. Here we extend the solution onto cases of total sliding and contact loss which is possible while using the displacement-driven formulation. The method requires the knowledge of the normal contact response obtained in our case as a result of statistical consideration of roughness of contact faces.

1. Introduction

Planar defects in solids (e.g. cracks, delaminations, debondings, imperfect intergranular contacts) represent internal contacts that, once activated by an ultrasonic wave, can produce contact acoustic nonlinearity [1–9]. Therefore, to detect, quantify or image such defects, nonlinear Non-Destructive Testing and Evaluation (NDT&E) methods are required. Modern NDT or NDE is often accompanied and supported by numerical modeling that helps to link the measured ultrasonic signals to the geometric or the physical parameters of the defects, thus achieving the final NDE objective.

Over the past decades, several models have been proposed to explain nonlinear wave-crack interactions [10]. These models often consist of two components: (1) a wave propagation model responsible for solving the elasticity equations in a material and (2) a crack model that supplements the elasticity equations with boundary conditions imposed at the internal crack boundaries by taking into account the specific dynamic behavior of cracks under normal and tangential loading. The crack models can be divided into two classes: the phenomenological models [1,11,12] and the physical

* Corresponding author.
E-mail addresses: Vladislav.Aleshin@iemn.univ-lille1.fr (V. Aleshin), Steven.Delrue@kuleuven.be (S. Delrue), olivier.boumatar@iemn.univ-lille1.fr (O. Bou Matar), Koen.VanDenAbeele@kuleuven.be (K. Van Den Abeele).

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models [6,13,14]. The first class of models tries to simulate the desired nonlinear behavior by means of simple (or more complex) stress-strain relations. These models can only be used for imitating contact nonlinearity in a qualitative way and not for explaining its physical behavior. The physical models, on the other hand, are more realistic, as they assume that there are physical mechanisms responsible for the nonlinear behavior of the defects. A simple example of a physical model for a crack with friction would consist of considering a zero-thickness slit with flat surfaces for which the Coulomb friction law is postulated. However, Coulomb's friction law does not provide a direct link between stresses and displacements. Indeed, in the stick regime, no sliding occurs and therefore no tangential displacement is possible by definition. In the sliding regime, however, the tangential displacement is not defined in the framework of Coulomb's friction law and has to be determined from external conditions.

An important obstacle that frictional models face is thus related to the fact that the traditional implementation of the Coulomb friction law written for loads is not sufficient to provide displacements for all contact states and, moreover, does not accept all possible load values either. Indeed, the tangential load for instance cannot exceed the normal one multiplied by the friction coefficient. However, for implementation of contact interactions, it is highly desirable to have an explicit link between loads and displacements which holds for any combination of the drive parameter values. Therefore, in this paper, we propose a crack model that is created using physics-based theoretical considerations that do not require any of the above mentioned additional assumptions. This is done by considering crack surfaces with a nontrivial topography (e.g. roughness) which will allow one to deduce a displacement-driven counterpart of the Coulomb friction law that meets the above mentioned requirement. Certainly, accounting for roughness makes the description more complicated since it results in the appearance of a partial slip regime where some parts on the contact area experience slip and some do not. Yet, this regime can be successfully described using the previously developed Method of Memory Diagrams (MMD) [15]. The novelty of the present approach consists in extension of the MMD onto the two other contact regimes that can appear when a pair of surfaces are brought to contact: contact loss and full sliding. This derivation is possible in the case where the contact system is driven by displacements. Note, however, that, since the model entirely results from the Coulomb friction law, it is quasi-static, meaning that all rate-dependent phenomena are ignored.

This paper focuses on the physics-based analytical description of a displacement-driven constitutive crack model that takes into account three different contact regimes: partial slip, total sliding and contact loss. Section 2 consists of a detailed description of the developed crack model. We start by explaining why traditional force-driven models of a crack with flat faces and Coulomb friction result in iterative data exchange procedures, whereas displacement-driven crack models do not. Furthermore, we discuss how such a displacement-driven system can be developed by introducing roughness in the model. As stated before, the proposed model is mainly based on the Method of Memory Diagrams, that was already developed by the authors to describe partial slip for rough surfaces in contact [15]. For the sake of clarity, the most important concepts about the earlier developed MMD algorithm will be repeated here, but now reformulated for the desired displacement-driven system. Moreover, we explain how the earlier developed MMD formulation can be extended to also take into account the two other contact regimes of total sliding and contact loss. In Section 3, we illustrate the outcome of the full algorithm by means of a numerical example. Illustrations on how the final developed crack model can be applied to describe the nonlinear interaction of elastic waves with cracks are subject to part II of the paper [16], where it is described how to introduce the crack model into the finite-element based, commercially available software package COMSOL Multiphysics® [17].

2. Crack model formulation

2.1. Force-driven and displacement-driven crack models

An accurate description of a crack model requires the concept of a small representative volume of material intersected by a crack. These volumes are often referred to as mesoscopic cells in which specific boundary conditions at the crack surfaces need to be defined. The term "mesoscopic" means that the cell size is much less than a typical wavelength or than any other characteristic scale of the macroscopic elastic field, but at the same time, the cell can host a large number of microscopic features (e.g. asperities present on crack surfaces). The mesoscopic level description includes average stresses and displacements, which are linked to each other by way of a particular relationship. The normal and tangential displacements (denoted here a and b), and normal and tangential stresses (N and T, respectively) are considered as lumped variables, in contrast to local (microscopic) stresses and displacements which are field variables. It is convenient to refer to N and T as forces per unit of nominal contact area, or just forces, in order to easily distinguish between the microscopic field variables and the mesoscopic lumped ones. In fact, "forces N and T" are traditionally used in contact mechanics since 1950s [18].

The objective of a crack model is to provide boundary conditions that represent a link between forces and displacements at crack faces. From the point of numerical simulations, the crack model should exchange force-displacement data with a solid mechanics unit that solves the elasticity equations in the bulk volume. Generally, both the crack model and the solid mechanics unit can be force- or displacement-driven. A situation where the crack model is driven by forces and the solid mechanics drive parameters are displacements is illustrated by the scheme displayed in Fig. 1. In particular, consideration of flat crack surfaces with friction and without adhesion typically results in such a solution which has an implicit character. This is due to the following reasons. If we consent that $N > 0$ for compression and that $a < 0$ for an open crack whose faces do not touch each other in a particular mesoscopic cell, then, for the normal interaction, two situations are possible: (1) $N > 0$ when the crack is closed (full contact in that cell) and therefore $a = 0$ and (2) $N = 0$ when the crack is open and a is undefined. Indeed, as the absence of interaction (i.e. $N = 0$) corresponds to any crack opening $a < 0$ and the actual displacement a therefore has to be determined from external conditions. The situation where $N < 0$ is not possible, since no attraction force between the faces exists in the absence of adhesion. An analogous behavior occurs for the tangential interaction. According to Coulomb's friction law, $|T| < \mu N$ means stick and therefore the tangential displacement b does not change. In the case where $|T| = \mu N$,

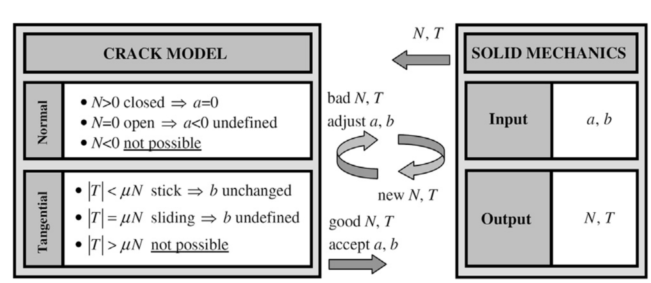

Fig. 1. A force-driven model of a crack with flat faces and Coulomb friction engenders an iterative data exchange procedure.