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Micro-foundation for a constant elasticity of substitution production function through mechanization

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ABSTRACT

We consider an increase in the range of capital use as a form of mechanization. A constant elasticity of substitution (CES) production function is dynamically derived from Leontief production functions through the endogenous complementary relationship between capital accumulation and mechanization. This implies that a CES production function can be resolved into technological change that does not involve changes in total factor productivity. Furthermore, using the normalizing procedure of the CES production function developed by de La Grandville [de La Grandville, O., 1989. In quest of the Slutsky diamond. American Economic Review 79, 468–481], we investigate how mechanization is related to the elasticity of substitution in our endogenous growth model.

1. Introduction

A constant elasticity of substitution (CES) production function was mathematically derived by Arrow et al. (1961) to consider various elasticities of substitution between capital and labor. The CES production function has played an important role in understanding economic growth. For example, de La Grandville (1989), Klump and de La Grandville (2000), Klump and Preissler (2000), Miyagiwa and Papageorgiou (2003, 2007) used it to investigate the relationship between economic growth and the elasticity of substitution between capital and labor. Duffy and Papageorgiou (2000) and Masanjala and Papageorgiou (2004) applied it empirically to explain cross-country variations in economic growth.

Apart from the definition of the elasticity of substitution between labor and capital, though, does elasticity of substitution have any economic meaning? Klump and Preissler (2000) discussed some of its plausible determinants. In the present paper, we dynamically derive a CES production function from Leontief production functions through an endogenous increase in the range of capital use; i.e., a kind of mechanization. By considering a complementary relationship between capital accumulation and mechanization, we can relate the difficulty of mechanization to the elasticity of substitution in our endogenous growth model.

In addition, a CES production function including a Cobb–Douglas production function has generally been used to measure changes in total factor productivity (TFP) of industries or economies. Many studies such as those in endogenous growth

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theories have considered technological change to explain changes in TFP. However, there would be little theoretical justification for considering only changes in TFP as technological change. Where does a CES production function itself come from? The present paper shows that a CES production function can be resolved into technological change characterized by mechanization, while its technological change does not appear in changes in TFP.

We assume that goods are produced through production-process steps. Because we can differentiate between steps in which capital predominates and those in which labor predominates, we consider that only capital or only labor is the input in each step. We term the continuum of steps in which capital is used as "the range of capital use". An increase in the range of capital use means mechanization.

Let us assume Leontief production functions as short-run production functions. Once a technology has been chosen, then even if the input of a step for the production process is increased, the output will not increase because the other steps then become a bottleneck. That is, while we can choose the appropriate technology depending on the factor prices *ex ante*, there would be no substitutability between capital and labor *ex post* because the technology would correspond to a fixed capital/labor ratio. This idea is essentially the same as that underlying putty-clay models. Therefore, our assumption of Leontief production functions as short-run production functions is reasonable. We derive the long-run production function as the locus of short-run production functions through a complementary relationship between capital accumulation and mechanization.[1] A long-run production function describes the maximum amount of output that a firm can produce from a particular technology set.

We first emphasize some important points. Assuming that fixed coefficients follow a Pareto distribution, Houthakker (1955–1956), in his analysis of activity, derived a Cobb–Douglas production function as the aggregation of micro-production functions. Levhari (1968) investigated the distribution for fixed coefficients among firms that yields a macro CES production function from micro-production functions. However, there was little explanation about the economics involved in these papers. Jones (2005), on the other hand, considered appropriate technologies in which an idea corresponds to a technology for combining capital and labor to produce output. Assuming that ideas follow a Pareto distribution, he presented a micro-foundation for a standard production function that exhibits steady-state growth. While we also consider appropriate technologies, we derive a complementary relationship between capital accumulation and mechanization. We then dynamically derive a CES production function in which the elasticity of substitution takes any value.[2] Therefore, we can present a micro-foundation for a CES production function in our endogenous growth model.

Assuming a trade-off in the distribution parameters of a CES aggregate where skilled and unskilled labor are imperfect substitutes, Caselli and Coleman (2006) found that in a given economy, an appropriate technology is chosen depending on the ratio of skilled to unskilled workers. We consider a trade-off between a capital-output coefficient and a labor-output coefficient through the range of capital use. This makes it possible to relate the difficulty of mechanization to the elasticity of substitution.

Because our technological change characterized by mechanization is represented by changes in capital per labor unit in equilibrium, we can propose technological change that essentially differs from technological progress in endogenous growth theories such as those conceived by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Technological progress in their models can be represented by an upward shift in the production function that is not measurable by changes in the quantities of input labor and capital. We, on the other hand, extract technological change from changes in input quantities.

The rest of the paper is organized as follows. Section 2 explains our model. Section 3 presents the micro-foundation for a CES production function. Section 4 concludes our paper with a brief summary of the main points.

2. Model

In a closed economy, there are many households and firms that are perfectly competitive. Goods are used for consumption and to accumulate capital.

2.1. Firms

We assume the following Leontief production function:

$$Y_t = \min\{f(a_t)K_t, g(a_t)L_t\}, \qquad f(a_t),\ g(a_t) > 0, \quad a_t \in [0,1], \tag{1}$$

where Y_t is the output of goods at time t, a_t is the range of capital use at time t, and $1 - a_t$ is therefore the range of labor use at time t, K_t is the input for capital, and L_t is that for labor at time t. Here, we call $f(a_t)$ and $g(a_t)$ the "capital-output coefficient" and "labor-output coefficient", respectively.

[1] Mechanization in our model is essentially the same as that of Zeira (2006). He explained economic growth through industrialization where machines replaced workers in a growing number of tasks. Zeira (1998) also authored a pioneering work on formalizing technological changes that replaced workers by machines.

[2] By considering a complementary relationship between capital accumulation and mechanization in a centralized economy, Nakamura and Nakamura (2008) derived a CES production function as the envelope of Cobb–Douglas production functions.