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Impulsive synchronization of discrete-time networked oscillators with partial input saturation[☆]

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ABSTRACT

In this paper, a novel impulsive control with partial input saturation is proposed to synchronize a class of discrete-time delayed coupled networks. The proposed impulsive control with partial input saturation means that the considered impulsive controllers are subject to input saturation, and only a fraction of nodes are injected with controllers. For this purpose, the Razumikhin-type technique and mathematical induction are utilized to derive a sufficient criterion. In addition, the control gains are calculated by transforming the criterion into a convex optimization problem. Finally, the result is verified through numerical simulation.

1. Introduction

Over the past decades, the study on discrete-time dynamics of oscillators has attracted many interests in the research fields due to the favorable prospects of digital implementations [1–7], e.g., system identification, image processing and optimization problems. As a common prerequisite condition of above systems, all coupled oscillators usually show a final state consistency when time evolves [1,8]. This phenomenon is called the synchronization, which has an important impact on both experimental and fundamental sciences, e.g., the self-organization and cooperation behaviors of neural firing in the brain activities [9]. Actually, synchronization of coupled oscillators originated from hypothesis and analysis of dynamical systems. Recently, there are some fruitful works focusing on this topic [4,5,8,9].

With the development of control theory, numerous approaches have been taken into account for synchronization of oscillators, e.g., adaptive control, pinning control, sampled-data control, impulsive control and, etc [8,10–21]. Among them, impulsive control is an important topic, and has been considered in various issues of dynamical systems [22–25]. Generally, impulsive control provides a scheme to apply a system with the control inputs only acting on a quite sparse sequence of time [26]. In engineering, there are many applications, including the instantaneous and intermittent impacts in mechanical

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systems, or the electron tunneling effects in microdevices [25]. Hence, impulsive control not only possesses a simple and effective structure, but also has gained and extended popularity in synchronization of networks [10,19–21,27–31]. For instance, in [28], synchronization analysis of networks with disconnected topologies has been tackled by using impulsive control. An impulsive controller, which is subject to control input delays, has been applied to dealing with the synchronization problem of stochastic neural networks in [27]. The pinning impulsive synchronization has been examined for nonlinear stochastic dynamical complex networks in [29].

Moreover, the saturation problem is one of the most common nonlinearities in practical implementations, since the capability of hardware devices may be limited to a finite range [32–34]. For instance, the activation function of neural networks like $\tanh(\cdot)$ is a saturated function, which can directly affect the dynamics of addressed networks. As mentioned in [33], when the effect of saturation is neglected, it may lead to performance degradation of networks. To describe the saturation problem among networks, there are two mainstream approaches in current works [32–37]: one is to apply a sector bound nonlinearity that constrains the control input, and the other is to find a polytopic representation in order to consider the bounded control input [35]. For instance, based on a sector bound nonlinearity, the nonlinearities arising from actuator saturation have been introduced in dynamics analysis of coupled neural networks in [35]. In [34], the semi-global consensus problem of multi-agent systems has been examined by utilizing low gain feedback, in which saturation input is modeled by a kind of polytopic representation.

However, there are distinct physical limitations in a same system, which means that saturation only exists in a part of dimensions or states, e.g., yaw and roll velocity in the dynamics of vehicles without constraints [36]. Particularly, the oscillators in network evolve independently and influence each other. Thus, these oscillators can be regarded as subsystem with different properties that are limited by the physical hardware. In this case, a part of the oscillators may be subject to input saturation. Some initial works on the partial input saturation problem have been addressed in [36,37]. Up to now, although the impulsive control has been widely recognized, the consideration of saturation into the impulsive controller design is still open, primarily due to the difficulty in mathematical analysis in tackling both input saturation and impulsive dynamical system. Moreover, the impulsive control with partial input saturation, in which only a part of dimensions or states are subjected to saturation, has not been addressed before.

In this paper, we tackle the synchronization of delayed coupled discrete-time networks via partial input-saturated impulsive control. The Razumikhin-type techniques are utilized to derive a sufficient synchronization criterion, which can be shown as certain linear matrix inequalities (LMIs). Subsequently, these matrix inequalities are transformed into the optimization problems so as to obtain the control gain matrix. Different from the existing works, the mainly contributions of this paper are shown in the following: (1) A novel impulsive control approach has been proposed, which takes into account partial input saturation. (2) To handle the partial input saturation, the Razumikhin-type techniques and the type of sector criteria by involving partial input saturation in [36,37] are combined to tackle the synchronization.

Notations: Throughout this brief, $\mathbb{R}^{n\times m}$ and $\mathbb{R}^n$ denote the set of $n\times m$ real matrix and the n-dimensional Euclidean space, respectively. The notation $X\geq Y (X>Y)$, where X and Y are symmetric matrices, means that $X-Y$ is positive semidefinite (positive definite). $\mathbb{N}=\{0,1,2,\ldots\}$ and $\mathbb{N}_{-\tau}=\{-\tau,-\tau+1,\ldots,-1,0\}$. I_n denotes n-dimensional identity matrix. $\|A\|$ is the norm of the matrix $A\in\mathbb{R}^{n\times n}$ by the Euclidean vector norm. For a given integer τ, let $D=\{\varphi:\mathbb{N}_{-\tau}\to\mathbb{R}^n\}$, we define $\|\varphi\|_\tau=\sup_{\theta\in\mathbb{N}_{-\tau}}\|\varphi(\theta)\|$. Moreover, the mathematical symbol $*$ is utilized to express a matrix that can be concluded by symmetry.

2. Problem formulations

In this paper, the network composed of N-identical delayed coupled oscillators with discrete-time dynamics is given below:

$$x_i(k+1)=Cx_i(k)+\widetilde{f}(x_i(k))+\nu\sum_{j=1}^{N}l_{ij}x_j(k-\tau_k), \tag{1}$$

where $x_i(k)\in\mathbb{R}^n$ means the state of the ith oscillator; τ_k is time-varying delay and satisfies $0\leq\tau_k\leq\tau$; ν is the coupling strength; the symmetric matrix $L=[l_{ij}]_{N\times N}$, i.e., coupling configuration matrix, stands for the topological structure of network, and is represented as follows: $l_{ij}>0\ (j\neq i)$ if there exists a communication link from oscillator j to i, otherwise $l_{ij}=0$, moreover, the diagonal entry of matrix L satisfies

$$l_{ii}=-\sum_{j=1,j\neq i}^{N}l_{ij};$$

if for any $\bar{x},\bar{y}\in\mathbb{R}^n$, there exists a scalar $\beta>0$ and a matrix $P>0$ satisfying the following condition, then the nonlinear function $\widetilde{f}(\cdot)\in\mathbb{R}^n$ is said to be Lipschitz [6]

$$[\widetilde{f}(\bar{x})-\widetilde{f}(\bar{y})]^TP[\widetilde{f}(\bar{x})-\widetilde{f}(\bar{y})]\leq\beta^2(\bar{x}-\bar{y})^T(\bar{x}-\bar{y}). \tag{2}$$

For $\theta\in\mathbb{N}_{-\tau}$, the network in (1) is supplemented with the initial conditions given by $x_i(k_0+\theta)=\phi_i(k_0+\theta)$.

Denote $s(k+1)$ by the solution of an individual oscillator of network in (1)

$$s(k+1)=Cs(k)+\widetilde{f}(s(k)), \tag{3}$$