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Persistent impulsive effects on stability of functional differential equations with finite or infinite delay[☆]

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ABSTRACT

In this paper, the stability problem of impulsive functional differential equations (IFDEs) is considered. Several criteria ensuring the uniform stability of IFDEs with finite or infinite delay are derived by establishing some new Razumikhin conditions. Different with the current existing ones, our development results can be applied to delay systems with persistent impulsive effects. Finally, an example and its computer simulation is given to show the effectiveness and advantages of the proposed method.

1. Introduction

The theory of impulsive functional differential equations (IFDEs) has been extensively investigated and developed by a large number of researchers due to their potential applications in many fields such as control technology, communication networks, and biological population management, etc, and many interesting results have been presented in the literature, see [1–10]. In particular, stability problem is one of the important fundamental theory in the investigation of IFDEs and forms an important class as regards their real applications [11–14]. Generally, the existing stability results for IFDEs can be classified into two groups: impulsive perturbation and impulsive stabilization. In the case where a given equation without impulses is stable, and it can remain the stability behavior under certain impulsive interference, it is regarded as impulsive perturbation problem [15–24], which can be illustrated by the following example.

Example 1.1. Consider a simple IFDE:

$$\begin{cases} \dot{x}(t) = -1.5x(t) + x(t-1), & t \geq 0, \\ x(k) = (1 + \frac{3}{k^2})x(k^-), & k \in \mathbb{Z}_+. \end{cases} \tag{A}$$

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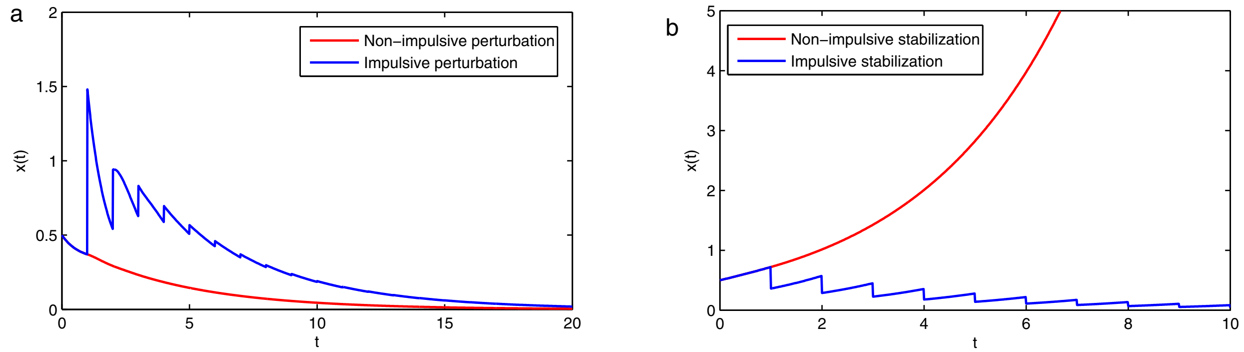

Fig. 1. (a) State trajectory of system (A) with or without impulsive perturbation. (b) State trajectory of system (B) with or without impulsive stabilization.

Obviously, system (A) is stable when there is no impulsive effects, and the stability behavior still can be guaranteed under above impulsive perturbations, which can be obtained by the results in [15]. The corresponding numerical simulation is shown in Fig. 1(a).

In addition, in the case where a given equation without impulses is unstable or stable, and it can be turned into uniformly stable, uniformly asymptotically stable even exponentially stable under proper impulsive control, it is regarded as impulsive stabilization problem [25–33]. This problem can be illustrated by the following example.

Example 1.2. Consider an IFDE:

$$\begin{cases} \dot{x}(t) = 0.2x(t) + 0.2x(t-1), & t \geq 0, \\ x(k) = 0.5x(k^-), & k \in \mathbb{Z}_+. \end{cases} \tag{B}$$

Obviously, system (B) is unstable when there is no impulsive effects, but it becomes stable under proper impulsive control, which can be obtained by the results in [25]. The corresponding numerical simulation is shown in Fig. 1(b). It implies that impulses may contribute to system dynamics.

Although many interesting results [15–33] on stability of IFDEs have been reported from impulsive perturbation or/and impulsive stabilization points of view, there are still many disadvantages. For instance, from impulsive perturbation point of view, most of existing results have a common assumption that $\Sigma\beta_k < \infty$ to guarantee the stability behavior, see above Example 1.1 and Refs. [15–24]. In other words, there is almost no impulsive effects at infinity, which is an unrealistic problem in real applications. As we know, a real system is often subjected to instantaneous perturbations and experience abrupt changes at certain moments of time, and the perturbations may be persistent or periodic. In biological neural networks, for example, when a stimuli from the body or the external factors is received by receptors in a persistent or periodic environment, the electrical impulses will be conveyed to the neural net and the persistent or periodic impulsive effects arise naturally in the net [34]. In such cases, those existing results [15–24] with the assumption that $\Sigma\beta_k < \infty$ will be infeasible. Additionally, from impulsive stabilization point of view, one may observe that most of existing results have a common assumption that $V(t_k) < V(t_k^-)$ at each impulse point t_k to stabilize an unstable systems with finite delay or infinite delay, see above Example 1.2 and Refs. [25–33]. This is also unrealistic in some real applications. Considering an cooperative ecosystem with impulsive harvesting and impulsive planting [35,36], it is known that $V(t_k) < V(t_k^-)$ at each impulsive harvesting time and $V(t_k) > V(t_k^-)$ at each impulsive planting time. In such cases, one of the practical problem is that how to control the impulsive effects such that the ecosystem can withstand those disturbances and keep persistence over a long period of time. As mentioned above, techniques and methods for stability problem of IFDEs should be further developed and explored.

The purpose of the present work is to improve the existing results [15–33] on stability of IFDEs with finite delay or infinite delay. By establishing some new Razumikhin conditions, the criteria ensuring the uniform stability of IFDEs are derived, which can be applied to delay systems with persistent impulsive effects. The rest of this paper is organized as follows. In Section 2, we first recall some preliminaries on IFDEs. In Section 3, we state the main results of the present paper, which are different from the existing ones. In Section 4, an example and its computer simulation is given. At last, we draw a conclusion in Section 5.

2. Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of positive real numbers, $\mathbb{Z}_+$ the set of positive integers and $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm $|\bullet|$. The impulse times t_k satisfy $0 \leq t_0 < t_1 < \cdots < t_k \to \infty$ as $k \to \infty$. For any interval $J \subseteq \mathbb{R}$, set $S \subseteq \mathbb{R}^k (1 \leq k \leq n), C(J, S) = \{\varphi : J \to\ S \text{ is continuous}\}$ and $PC(J, S) = \{\varphi : J \to\ S$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+)$, $\varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$. Let $\mathbb{C}_\alpha$ be an open set