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Input-to-state stability of impulsive systems and their networks

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ABSTRACT

This paper considers input-to-state stability (ISS) characterization for a class of impulsive systems which jump map depends on time. We provide sufficient conditions in terms of exponential ISS-Lyapunov functions equipped with an appropriate dwell-time condition for establishing ISS property. Some modifications of dwell times which are more conservative, but easy to be verified are being introduced. We also show that impulsive system with multiple jump maps can naturally represent an interconnection of several impulsive systems with different impulse time sequences. Then we present a procedure to verify ISS of such networks.

1. Introduction

Impulsive systems describe processes that combine continuous and discontinuous behaviour. A variety of examples can be found in mathematical modelling of applications in logistics, robotics, population dynamics, etc. A basic mathematical theory of impulsive systems as well as fundamental results on existence and stability of solutions can be found in [1,2] and references therein.

Nearly at the same time the concept of input-to-state stability (ISS) for systems of ordinary differential equations has been introduced by Sontag [3]. ISS characterizes a behaviour of solutions with respect to external inputs. Later it was also studied for discrete-time systems [4], switched systems [5], and hybrid dynamical systems [6].

ISS properties of impulsive systems were firstly studied by Hespanha et al. in [7]. The authors provided a set of Lyapunov-based sufficient conditions for establishing ISS with respect to suitable classes of impulsive time sequences. The same approach was used in [8] to justify integral ISS property of impulsive systems. The mentioned results were obtained by introducing the concept of exponential ISS-Lyapunov function for impulsive system. Two constants that are called rate coefficients were used to characterize a behaviour of ISS-Lyapunov function along the trajectories of impulsive system during flows (constant c) and impulsive jumps (constant d). The positive values of rate coefficients correspond to the case of positive impact of flow/jumps onto ISS property, and vice versa, if c (or d) is negative it means that the corresponding flows (or jumps) play against stability. The relation called dwell-time condition (DTC) that restricts the frequency of impulsive jumps in order to guarantee ISS of impulsive systems has been introduced.

A motivation for this paper is the following. Often in real applications impulsive jumps can be of different types and impact ISS property in different manners (see Example 1): some jumps can contribute towards stability, and others can play

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against it. This requires a more flexible tool that accounts different types of influences caused by different parts of system in a precise manner. In this paper we introduce a notion of a candidate exponential ISS-Lyapunov function with several rate coefficients that describe system's dynamics on jumps and develop a new dwell-time condition that guarantees ISS property in such a case. Also we present a new modification of dwell-time condition that is applicable to a sufficiently wide class of systems in which impulsive time sequences converge to periodic or quasiperiodic regimes.

An important question of ISS theory is establishing sufficient conditions for ISS of interconnected systems. The first results on the ISS property were given for two coupled continuous systems in [9] and for an arbitrarily large number ($n \in \mathbb{N}$) of coupled continuous systems in [10]. Lyapunov versions of these so-called ISS small-gain theorems were proved in [11] (two systems) and in [12] (n systems). Small-gain theorems for impulsive systems with and without time-delay were established in [13]. A complementary result for infinite-dimensional impulsive systems were developed in [14].

The latest developments in the area of interconnections and systems of a large scale were made in a strongly related class of hybrid systems. We refer the reader to papers of [15–18] with the most recent small-gain theorems on ISS of hybrid systems. However, solutions to hybrid systems are defined on hybrid time domains, as opposed to the usual time defined on the real line. As it was mentioned in [8], this leads to a distinct notion of ISS, and some systems that are ISS in impulsive framework are not ISS in the hybrid framework. This motivates an importance of investigation of explicitly impulsive systems apart from hybrid ones.

A significant lack of the previously developed results on interconnections of impulsive systems is that impulsive time sequences of each subsystem should coincide in order to apply known results. In this paper we prove that a network of impulsive systems with different impulsive time sequences can be represented in the form of impulsive system with multiple jumps maps. This enables to employ previously developed methods [13,14] for an ISS-Lyapunov function construction. Combining this with a newly presented dwell-time condition enables a comprehensive stability analysis of interconnections in which subsystems have different impulse time sequences.

The first step in this direction was made in the conference paper [19] where the concept of exponential ISS Lyapunov function with multiple rate coefficients was introduced. However there we provide theorem on sufficient Lyapunov-like conditions for ISS without proof. Here we prove it rigorously and moreover present a number of alternative dwell-time conditions that are easy to be verified. Additionally in this paper we propose a procedure for ISS analysis of interconnection of an arbitrary number of subsystems with nonlinear gain functions and present an appropriate example. These essentially extend the results from [19] which treats the case of two subsystems with linear gains only.

The rest of the paper is organized as follows. Section 2 contains a short motivation to consider impulsive systems with multiple jump maps. A Lyapunov-like sufficient conditions for ISS of such impulsive system along with an appropriate dwell-time condition are presented in Section 3. Different types of dwell-time condition and its application in particular cases are discussed in Section 4. A procedure of ISS analysis of a network of impulsive systems with different impulse time sequences is presented in Section 5. A discussion and a short conclusion complete the paper.

2. Motivation

Well-known ISS results [8,13,14] for impulsive systems were developed for a system of the type

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)), & t \neq t_k, \quad k \in \{1, 2, \dots\}, \\ x(t) = g(x^-(t), u^-(t)), & t = t_k, \quad k \in \{1, 2, \dots\} \end{cases} \tag{1}$$

where $\{t_1, t_2, t_3, \dots\}$ is a strictly increasing sequence of impulse times in (t_0, ∞) for some initial time t_0; the state $x(t) \in \mathbb{R}^N$ is absolutely continuous between impulses; input $u(t) \in \mathbb{R}^M$ is locally bounded and Lebesgue-measurable function; f and g are functions from $\mathbb{R}^N \times \mathbb{R}^M$ to $\mathbb{R}^N$, with f locally Lipschitz. The state x and the input u are assumed to be right-continuous, and to have left limits at all times. We denote by $(\cdot)^-$ the left-limit operator, i.e., $x^-(t) = \lim_{s \nearrow t} x(s)$. However system (1) does not cover a variety of many real processes. Consider the following

Example 1. Let the number $x \in \mathbb{R}_+$ of goods in a storage be continuously decreasing proportionally to the number of items with rate coefficient 0.2. But every odd day ($\mathbb{T}_1 = \{1, 3, 5, \dots\}$) a delivery truck doubles the number of items, and every even day ($\mathbb{T}_2 = \{2, 4, 6, \dots\}$) a delivery truck takes out 40% of items. The evolution of this process can be modelled as

$$\begin{cases} \dot{x}(t) = -0.2x(t), & t \notin \mathbb{T}_1 \cup \mathbb{T}_2, \\ x(t) = \begin{cases} 2x^-(t), & t \in \mathbb{T}_1, \\ 0.6x^-(t), & t \in \mathbb{T}_2, \end{cases} \end{cases} \tag{2}$$

System (2) does not fit into the class of systems (1) because its jump map depends on time. Moreover, some jumps contribute towards stability, and the others play against it. Numerical simulations show that a trivial solution to this system is likely to be globally asymptotically stable (see Fig. 1).

Impulsive systems with time-dependent jump map have been considered in [20–22]. In these works, authors study stability properties of a wide class of impulsive systems which additionally include switchings of continuous part, multiple jump maps, and delays. The focus of this paper is just on multiple jump maps. We provide a more general result comparing to [20–22] only regarding this one aspect by allowing both stable and unstable impulsive jumps and an introduction of a new dwell-time condition that balances all the stable and unstable dynamics to achieve stability.