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AGGREGATION ISSUES IN THE ESTIMATION OF LINEAR PROGRAMMING PRODUCTIVITY MEASURES

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This paper demonstrates the sensitivity of the linear programming approach in the estimation of productivity measures in the primal framework. Specifically, the sensitivity to the number of constraints (level of dis-aggregation) and imposition of returns to scale constraints is evaluated. Further, the shadow or dual values are recovered from the linear program and compared to the market prices used in the ideal Fisher index approach. Empirical application to U.S. state-level time series data from 1960-2004 reveal productivity change decreases with increases in the number of constraints. Divergence in productivity measures is observed due to the choice of method imposed, various levels of commodity/input aggregation, and technology assumptions. Due to the piecewise linear approximation of the nonparametric programming approach, the shadow share-weights are skewed leading to the difference in the productivity measures due to aggregation.

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I. Introduction

The linear programming (LP) approach has gained popularity since the early 1990s due to its ability to impose little a priori functional form, handle multiple outputs-inputs without the need of price data, and accommodate weak and strong disposability assumptions. However, the LP approach, due to its piecewise linear approximation of the technology or theoretical frontier, is conditioned by the number of decision making units (DMU) and the number of constraints (in our case the level of input and output aggregation) in the model. The sensitivity of LP efficiency measures due to output and input aggregation has been established (Thomas and Tauer 1994; Tauer and Hanchar 1995; and Shaik 2007) and referred to as the "curse of dimensionality" problem (see, e.g., Thanassoulis et al. 2008: 320). The "curse of dimensionality" problem associated with an increase in the number of constraints (or level of disaggregation), leads to an increase or decrease in the number of reference points resulting in a decrease or increase in the efficiency and productivity measures. These aggregation issues have been addressed in the literature (Blackorby and Russell 1999; Färe and Zelenyuk 2003; and Simar and Zelenyuk 2003) with the use of dual input, output prices. However, explaining the aggregation issue in the primal framework without the explicit or implicit use of dual or shadow price is challenging.

This paper addresses the "curse of dimensionality" issue by demonstrating that the problem may be due to the shadow or dual values recovered from the constraints of the LP approach. The dual values of the LP constraints should reflect technology and economic behavior of individual DMUs (or states in this case). Theoretically (Caves, Christensen and Diewert 1982a and 1982b), the computation of productivity measures involves the use of market prices in the case of the ideal Fisher index approach, marginal product in the case of the parametric approach, and shadow or dual values in the case of LP approach. We also demonstrate the shadow or dual values recovered from the LP constraints depend on how the return to scale constraint is imposed in the estimation of the LP productivity measures. The input-based Malmquist productivity index (IMP) or output-based Malmquist productivity index (OMP) impose constant returns to scale (CRS) or variable returns to scale (VRS) restrictions simultaneously in the input and output constraints (see Färe et al. 1994; Färe et al. 1998; and Grifell-Tatje and Lovell 1995). In contrast, the Malmquist total factor productivity (MTFP) index model (see Bjurek 1996) imposes constant returns to scale independently in input and output constraints. Other advantages of the MTFP index (a Hicks-Moorsteen type index) over the standard Malmquist productivity