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Preparation and measurement in quantum memory models

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HIGHLIGHTS

- A reconsideration of preparation and measurement in quantum cognition.
- An interpretation of contextuality based on the preparation and measurement processes.
- New avenues for understanding cognition using quantum tomography and Neumark's theorem.

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ABSTRACT

Quantum Cognition has delivered a number of models for semantic memory, but to date these have tended to assume pure states and projective measurement. Here we relax these assumptions. A quantum inspired model of human word association experiments will be extended using a density matrix representation of human memory and a POVM based upon non-ideal measurements. Our formulation allows for a consideration of key terms like measurement and contextuality within a rigorous modern approach. This approach both provides new conceptual advances and suggests new experimental protocols.

1. Introduction

How should we model memory? As Shiffrin states:

None of the models we use in psychology or cognitive science, at least for any behavioral tasks I find to be of any interest, are correct. We build models to increase our understanding of, and to slightly better approximate, the incredibly complex cognitive systems that determine behavior. (Shiffrin, 2003)

However, this pragmatism raises an interesting point. What do our models of memory assume? And how do they limit the way in which we can formulate a given memory model?

Currently many models of Quantum Cognition (QC) apply a single state vector that assumes a system in a pure state (Aerts, 2011; Bruza, Kitto, Nelson, & McEvoy, 2009; Bruza, Kitto, Ramm, & Sitbon, 2015; Nelson, Kitto, Galea, McEvoy, & Bruza, 2013; Pothos & Busemeyer, 2013). However, when we perform memory experiments we obtain ensemble data for a collection of subjects. This cannot be modeled with a pure state, rather a mixed state is required. In this work we will make use of the density matrix representation to model ensembles of human subjects in word association experiments. At first, we will provide a detailed technical description of von Neumann projective valued measurement (PVM). Although PVM measurement has been used in QC before, especially in the recall experiment of Bruza et al. (2009), a better technical description is necessary to describe measurement on ensembles of subjects. Here we will make use of a more precise notation for the specific case of two observables in the recall experiment, and then we will generalize this notation for more possible senses. This will enable us to describe scenarios that have more possible outcomes for each observable.

Another limitation of previous models for semantic memory in QC centers around the usage of projective measurement for cognitive systems. This is highly restrictive because QC (i) does not necessarily assume an orthogonal relationship between operators, and (ii) sometimes entails violations of repeatability. An analysis of these restrictions associated with the PVM formalism will lead us to introduce the more modern and general positive operator valued measure (POVM). We will show that this non-orthogonal measurement provides new understanding and extensions of the standard advantages of quantum inspired models of memory.

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Some existing research has already applied POVM in the construction of quantum models of cognition. For example, Khrennikov, Basieva, Dzhafarov, and Busemeyer (2014) used POVM to model different arrangements of questions in opinion polling, including "response (non)replicability" and "question order effect". In another work, Khrennikov and Basieva (2014) employed POVM to describe a situation in which there are not sharp "Yes/No" answers to dichotomous decision observables. Yearsley (2017) and Yearsley and Busemeyer (2016) provide a detailed tutorial for using POVM to model noisy and imperfect measurement, and their structure was used by Denolf, Martínez-Martínez, Josephy and Barque-Duran (2017) to model the prisoner's dilemma experiment. During this period of emerging interest, we recognized that POVM could provide a natural model of the process of conceptual combination (Aliakbarzadeh & Kitto, 2016), and introduced a generalized Bell inequality, where POVMs were used to represent joint nonideal measurement for two observables. The current work will extend this early promising result, additionally describing a more general form of POVM for one observable.

We will show that the density matrix representation and POVM formulation suggest new sources of contextuality in the preparation and measurement processes respectively. This allows us to reconsider the interpretation of context within these new models. Although it is important to explain existing contextuality in cognitive experiments using better mathematical methods, we believe it is also essential to consider other sources for contextuality in those experiments; sources of contextuality that were ignored in previous work.

We also introduce another application of POVM in the modeling of memory. We will use Neumark's dilation theorem to relate the full cognitive state of a subject to a restricted substate which represents only those cognitive processes through which a subject participates in an experiment.

At the end of the paper, we will discuss a future direction that we believe will contribute to better understanding of cognitive states. We will point to a possible application in using Quantum Tomography to characterize the unknown state of a cognitive system. Using the insights that we gain from this characterization, we will suggest that a new experimental protocol could be created based on repeating projective measurements on similar ensembles of a subject to specify the unknown state of that subject. In an idealized situation, the whole parameters of an unknown cognitive state could be specified using a single POVM.

2. The quantum model of memory

Quantum Models of Memory (Bruza et al., 2009, 2015; Nelson et al., 2013) treat words as states in a Hilbert space. The combined activation of words in memory is modeled using an entangled state, where an associative network is either fully activated, or not. In Nelson et al. (2013), it was argued that associative semantic networks are constructed through the complex set of experiences that people undergo throughout their lifetime, and so are closely related to episodic memory, a point that opens up the possibility for linking episodic and semantic models of memory if we can construct more plausible relationships between them in our formalism. In particular, episodic memories beyond the boundaries of an experiment can be considered a form of experimental noise, a point that we will return to shortly.

Semantic associative models imply that the way in which a subject responds to a prime will affect their ability to recall other words not directly connected to that prime in e.g. a semantic network (Nelson, McEvoy, and Schreiber, 2004). This assumption can be tested experimentally, and in Bruza et al. (2015) a framework is provided for considering whether conceptual combination can be considered compositionally or not. Two tests are used to establish compositionality; Marginal selectivity (Dzhafarov & Kujala, 2012) and a Bell type inequality. A number (21 from a total of 24) of conceptual combinations in that paper violated the marginal selectivity condition, while one of the combinations (BATTERY CHARGE) appears to satisfy marginal selectivity but violates a Bell-type inequality. More data is required before these results can be considered definitive. However, at this juncture it is a good idea to reconsider the theoretical apparatus of that model. Its reliance upon standard quantum models leaves it open to a number of criticisms from the perspective of psychology. Indeed, for a number of reasons that will become apparent shortly we consider it important to extend that model to a more general and modern formulation. We will begin this extension with a move to the density matrix formulation.

2.1. Constructing a density matrix

We start with a consideration of the way in which a subject might recall an ambiguous word A when cued with a particular prime. In quantum memory models this prime is represented as a basis state (i.e. a measurement context). Here we will use the eventualities $\{a', a''\}$ to describe a subject's responses to a concept A, which can be interpreted according to one of two possible dominant and subordinate *senses*. When the dominant sense of concept A is primed, and A is interpreted in that sense by the human subject, then we designate $a' = +1$. If A is not interpreted in that sense after priming the dominant sense, then we write $a' = -1$. Similarly, $a'' = \{1, -1\}$ relates to situations where the subordinate sense of concept A primed, and either recalled $(+1)$ or not (-1).

An example will help to make this formalism clear. Consider an experimental protocol where a subject cued with a concept A (e.g. BOXER) using a word on a screen "boxer". According to the USF free association norms (Nelson et al., 2004), a subject is more likely to interpret BOXER in the sport sense than the animal sense. We term the sporting sense dominant and the animal sense subordinate. If a subject is first primed with the dominant sense of BOXER using the word "glove", and then asked to interpret the concept BOXER, there is high possibility that they will recall a word that has a sport sense. This measurement process is represented by $\mathbb{A}'$, the result given by the subject is represented with a', and $a' = +1$, as the response agrees with the way in which the subject was primed. If the subject interprets BOXER in another sense, then we write $a' = -1$. Conversely, if at first the subject is shown the word "vampire", then this is likely to awake the animal sense in the mind of human subject. When the subject responds in a way that agrees with the animal sense of the priming we write $a'' = +1$, but if the concept is not interpreted in this subordinate sense, we use $a'' = -1$.

Adopting von Neumann's approach to the quantum measurement of an idealized system using self-adjoint linear operators, we assume that an orthonormal basis exists. We can now construct a Hermitian matrix $\mathbb{A}$ as a series of projection operators (Bruza et al., 2009)

$$\mathbb{A} = \sum_k a_k \mathbb{P}_k \tag{1}$$

where $\mathbb{P}_k$ is the projector onto the eigenspace of $\mathbb{A}$ with eigenvalue a_k, and each a_k corresponds to the results of the measurement $\mathbb{A}$. As an example, for two eigenvalues a_1 and a_{-1}, we can rewrite the von Neumann measurement as

$$\mathbb{A} = a_1\mathbb{P}_1 + a_{-1}\mathbb{P}_{-1}, \tag{2}$$

where $\mathbb{P}_1$ and $\mathbb{P}_{-1}$ are the projectors onto the eigenspace of $\mathbb{A}$ for those two eigenvalues.