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Learning preferences from paired opposite-based semantics ☆

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ABSTRACT

Preference semantics examine the meaning of the preference predicate, according to the way that alternatives can be understood and organized for decision making purposes. Through opposite-based semantics, preference structures can be characterized by their paired decomposition of preference into opposite poles, and their respective valuation of binary preference relations. Extending paired semantics by fuzzy sets, preference relations can be represented in a gradual functional form, under an enhanced representational frame for examining the meaning of preference. Following a semantic argument on the character of opposition, the compound meaning of preference emerges from the fuzzy reinforcement of paired opposite concepts, searching for significant evidence for affirming dominance among the decision objects. Here we propose a general model for the paired decomposition of preference, examining its characteristic semantics under a binary and fuzzy logical frame, and identifying solutions with different values of significance for preference learning.

1. Introduction

The meaning of the preference concept refers to the way decision objects or alternatives can be ordered for decision making purposes. This concept can be examined through the preference predicate, regarding the paired opposite decomposition of the positive and negative perceptions of preference. In psychology (see e.g. [3,11,17]), the meaning of concepts has been studied in relation to its valuation as being either positive or negative, eliciting a subjective measurement from the individual, but at the same time, requiring that the individual somehow solves the natural ambivalence involved in understanding opposite perceptions. For doing this, a semantic scale has been commonly used (initially proposed in [17]), measuring the meaning of concepts according to a given pair of opposite poles.

In decision theory (see e.g. [4,10,22]), such a *bi-polarity* has been studied from two perspectives. The first one can be referred as the *univariate model*, introducing a one-dimensional scale with opposite references as endpoints, in such a way that one of these references is taken to be (or is understood) as positive, in opposition to the other which is considered as its negative counterpart. In this setting, it is possible to further introduce a reciprocity assumption, so that the verification status of one pole entails by complementation a particular status of the other. For example, if we assume reciprocal preferences and we have absolute preference for watching a fiction movie over a documentary, then it is already assumed

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that we have absolutely no preference for watching a documentary. In this (restricted) univariate reciprocal model, where opposite preference is directly associated with the inverse preference, it is then possible to prefer one or the other, and if no reciprocity is imposed, we may even prefer nothing at all, such that the value for preference is either positive, negative, or neither positive nor negative.

On the other hand, the second perspective on bipolarity can be referred as the *unipolar bivariate model* (see e.g. [10, 11]), where a concept can be positive, negative, neither positive nor negative, or both positive and negative, thus allowing preference (and aversion) for watching both a fiction and a documentary. Then the relation among poles is not simple, but rather complex and depends on the particular semantic relation holding among the opposite concepts. Even more, taking into consideration multi-dimensional concepts, like multiple viewpoints describing the properties of objects, the semantic relation holding among opposites may require a more complex analysis.

Focusing on the different neutral states holding in between the opposite poles, and stressing the determinant role that those states have for representing the meaning of concepts, *logical paired structures* [13] provide an adequate framework to address such complexity. Particularly, in previous works we have studied how the different neutral states in between opposite poles that are postulated in the context of paired structures may be useful tools to represent and understand the complexity of preference concepts (see [8]), allowing to configure a pertinent valuation preference structure for ordering the decision objects or alternatives.

Furthermore, paired structures [8,13] explicitly represent the different and non-reciprocal sources of information building up the complex meaning of preference. In this respect, and on a neurological level, it is observed that the meaning of concepts emerges from the multiple positive and aversive stimuli composing perceptions and emotions [3,9]. In this sense, different pleasant and unpleasant affective components of the same sensory stimulus, processed separately in different physical areas of the brain (see e.g. [16,23]), may provide the inputs of human behavior and decision making [1,9]. Therefore, as neurological observation suggests, the positive and negative counterparts are formed and evaluated separately, in an independent manner, configuring a *significant* decision space with respect to the available (positive and negative) evidence.

In this way, opposite sets of evidence can be simultaneously evaluated as separate entities, coming together under the *fuzzy reinforcement* of their intensities. That is, given the separate nature of positive and negative aspects, they can be jointly examined as they reinforce each other under an appropriate aggregation process based on *opposition operators* [13,18]. As it will be examined in detail in the later sections of this paper (Sections 5 and 6), opposite pieces of evidence, coming from different sources, can be used to reinforce each other in order to find greater significance to their inferred meaning.

Here we propose a general setting where the performance of the different preference models can be formally assessed according to their significance. For this purpose, a measure of relative significance is introduced for evaluating the amount of evidence for affirming preference. As a result, the aggregation process unravels while searching for significant evidence on pairwise dominance for preference learning and intelligent (automatic) decision support, where the reliance on the emotional meaning associated to the alternatives allows explaining and identifying satisfactory (descriptive) viewpoints for decision-making (see e.g. [12,22]).

In order to examine opposite-based preference semantics for preference learning, Section 2 introduces standard preference structures, where the inverse preference relation represents the negative perception on preference. Section 3 extends the analysis to more complex preference structures, where positive and negative preferences are independently represented and measured. Then, in Section 4, fuzzy preference structures are introduced, examining a general frame for fuzzy paired preference semantics. In Section 5, a proposal for the significance of preference orders is given, and in Section 6, the general methodology for learning preferences is proposed, based on fuzzy reinforcement and maximal significance. Finally, Section 7 offers a numerical example illustrating the proposed methodology, ending with some open challenges for future research.

2. Standard preference semantics

Given a set of alternatives $\mathbb{A}$, a crisp preference structure can be defined $\forall(a,b) \in \mathbb{A}^2$, by the decomposition of the weak preference predicate $R(a,b) = a$ *is at least as desired as* b, into three basic binary relations P, I, J, such that (see e.g. [5]),

- **P:** The pair of alternatives $(a,b) \in \mathbb{A}^2$ belongs to the *strict preference* relation P, if and only if a is more desired than b, expressed by $P(a,b)$. The inverse strict preference predicate, *b is more desired than a*, is expressed by $P(b,a) = P^{-1}(a,b)$.
- **I:** The pair of alternatives $(a,b) \in \mathbb{A}^2$ belongs to the *indifference* relation I, if and only if a is as much as desired as b, expressed by $I(a,b)$.
- **J:** The pair of alternatives $(a,b) \in \mathbb{A}^2$ belongs to the *incomparability* relation J, if and only if a cannot be compared with b, expressed by $J(a,b)$.

Under standard preference modeling (see again [5] but also [19] and [14]), it is assumed that $\forall a,b \in \mathbb{A}$, the relations I and J are symmetrical, such that $I(a,b) = I(b,a)$ and $J(a,b) = J(b,a)$ hold, I is reflexive, such that $I(a,a)$ holds, J is irreflexive, such that $J(a,a)$ does not hold, and P is asymmetrical, such that $P(a,b)$ and $P(b,a)$ cannot hold simultaneously. Then, the preference structure (P, I, J), is such that only one situation holds, as in,

$$P \cap I = \emptyset, \quad (1)$$

$$P \cap J = \emptyset, \quad (2)$$

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