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IFAC PapersOnLine 50-1 (2017) 9438–9443

Model and Algorithms for Competitiveness Maximization on Complex Networks ⋆

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Abstract: In this paper we study a competition model on complex networks, where two competing agents are fixed to different states while other agents are evolving to update their states through interactions according to a distributed consensus rule. We consider the situation where one competitor has the opportunity to add new links to other evolving agents such that it could improve its influence on the number of its supporters. We focus on the problem of how to add these new links in order to maximize the influence of a competitor against its rival, referred to as *competitiveness*. We formulate this competition as a competitiveness maximization problem, which tries to maximize the number of supports of a given competitor against its rival. We analyze the properties of this problem on some special graphs and provide optimal solutions for them, respectively. We design a simulated annealing algorithm and three heuristic algorithms to approximately solve this NP-hard constrained optimization problem.

Keywords: Competitive dynamics, simulated annealing algorithm, heuristic algorithm, complex network.

1. INTRODUCTION

Competition is everywhere in our real world, such as competition among languages (Patriarca et al. (2012)), retailers (Chiu et al. (2009)) and party leaders (Chatterjee et al. (2013)), to name just a couple. As a common social phenomenon, competition has attracted a lot of attention recently. One of the mathematical models for competition on complex networks was the modified voter model (Mobilia et al. (2007)), where zealots with opposite opinions were introduced to the classical voter model. Some models representing the competition among innovative products were constructed based on the word-of-mouth propagation model (Carnes et al. (2007); Fazeli and Jadbabaie (2012)). Modified SIS (Susceptible-Infected-Susceptible) models were proposed to investigate competitions among different kinds of viruses and epidemics (Beutel et al. (2012); Prakash et al. (2012)). A common feature of the above models is that agents only expressed discrete states: supportive or antagonistic, product A or product B, susceptible or infected. Very few models with continuous states were proposed to study competing situations. Acemoglu et al. (2013) investigated a gossip model with stubborn agents, where agents possessed continuous states, with long-run disagreements and persistent fluctuations appeared. Recently, we also proposed a model of competition with continuous states, where two competitors had different fixed states, and each other evolving agent adjusted its state according to a distributed consensus rule (Zhao et al. (2014)). Wherein we characterized the competition result as a function of the network structure and proposed a simple criterion for predicting the outcome.

Besides modeling competitions and analyzing the result of any given competing situation, here we further consider how to influence the competition result if we have an opportunity to manipulate some conditions of the competition. This might be more appealing to the competitors who are eagerly seeking strategies to win the competition, which is also closely related to the classical influence maximization problem. Roughly speaking, the influence maximization problem considers how to choose a set of nodes in a network as initial active ones such that they can maximize the spread of a message or an idea to the other agents in the network through their influence on others. Most of the existing works focused on the situation without competition, like the influence maximization

⋆ This work was supported by the National Natural Science Foundation of China under Grant Nos. 61374176, 61304158, 61473189, and 61503207, the Natural Science Foundation of Shandong Province (ZR2015PF003), the Project Funded by China Postdoctoral Science Foundation (2015M571996), the Qingdao Postdoctoral Application Research Project (2015123), the Science Fund for Creative Research Groups of the National Natural Science Foundation of China (No. 61221003), and the Hong Kong Research Grants Council under the GRF Grant CityU 11208515.

2405-8963
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.1463

problem in the word-of-mouth propagation model (Gionis et al. (2013); Kempe et al. (2003); Morone and Makse (2015)). The influence maximization problem was also studied under competition. In the situation of Yildiz et al. (2013), Fardad et al. (2012), and Carnes et al. (2007), given the locations of one set of competitors, the problem was to choose the other set of competitors' locations such that their relative influence was maximized.

All the above works, both with and without competition, investigated the problem of how to place the influential agents on a fixed network. Here, in this paper, the locations of the competitors in the network are fixed, yet one of the competitors could enhance its influence by changing the network structure (i.e., adding new links to connect other agents). Note that changing the network structure is resource-costing, so we suppose that the number of new links is limited. Then, we come to the problem of how to add these limited number of new links in order to maximize the competitor's competitiveness. We consider one competitiveness maximization problem, to maximize the number of supports of a given competitor against the other competitor. Though this problem is proved to be NP-hard in our previous work (Zhao et al. (2015)), we provide optimal solutions for some special graphs in this paper. We also design a modified Simulated Annealing algorithm and three heuristic algorithms based on network centralities to approximatively solve this problem. The performances of the proposed algorithms are demonstrated by simulations.

The paper is organized as follows. In Section 2, the competitiveness maximization problem is described. Section 3 provides several optimal link-adding solutions for some special graphs. Section 4 includes algorithm design and simulations to demonstrate their performances. Section 5 presents some concluding remarks.

2. PROBLEM FORMATION

2.1 A Competition Model

In this section, we introduce a competition model, which was studied by Zhao et al. (2014). We consider a network as a directed and unweighted graph $G = (V, E)$ with N agents and M links, where the agents are denoted by set $V = \{1, 2, \cdots, N\}$ and E is the set of edges between agents. A coupling matrix $A = (a_{kl})_{N\times N}$ is used to describe the topology of the network: if there is a link from agent k to agent l, which means agent k is directly influenced by agent l, then $a_{kl} = 1$; otherwise, $a_{kl} = 0$. For simplicity, assume that there are only two competitors in the network, denoted as agents i and j, which have different fixed states as follows:

$$x_i(t) \equiv +1, x_j(t) \equiv -1, \forall t \geq 0. \tag{1}$$

Except for the two competitors, every other agent is evolving, called a normal agent in this paper. The normal agent $k \in V\backslash\{i, j\}$ with a random initial state value in R updates its state as follows:

$$x_k(t+1) = x_k(t) + \varepsilon \sum_{l\in N_k} a_{kl}\left(x_l(t) - x_k(t)\right), \tag{2}$$

where $x_k(t)$ is the state of agent k at time t; $N_k = \{l \in V | a_{kl} = 1\}$ represents the neighborhood of agent k; the fixed parameter $\varepsilon > 0$ describes the level of neighbors' influence on agent k.

Reorder the agents such that the two competitors come the last. Thus, the coupling matrix A can be rewritten as

$$A = \begin{bmatrix} \bar{A} & \mathbf{c}_i & \mathbf{c}_j \\ \mathbf{r}_i & * & * \\ \mathbf{r}_j & * & * \end{bmatrix},$$

where $\bar{A} \in R^{(N-2)\times(N-2)}$ captures the neighboring relationship between normal agents; column $\mathbf{c}_i$ and row $\mathbf{r}_i$ represent the relationship between the normal agents and competitor i; column $\mathbf{c}_j$ and row $\mathbf{r}_j$ represent the relationship between the normal agents and competitor j. A diagonal matrix D, whose diagonal elements are the agents' out-degrees, is defined by

$$D = \begin{bmatrix} \bar{D} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & d_i & 0 \\ \mathbf{0} & 0 & d_j \end{bmatrix},$$

where diagonal matrix $\bar{D} \in R^{(N-2)\times(N-2)}$ is the out-degree matrix of normal agents, and d_i (d_j) is the out-degree of competitor i (j).

Then, a state updating rule is introduced as follows:

$$\begin{bmatrix} X_{norm}(t+1) \\ x_i(t+1) \\ x_j(t+1) \end{bmatrix} = \begin{bmatrix} Q & B \\ \mathbf{0} & 1\ 0 \\ \mathbf{0} & 0\ 1 \end{bmatrix} \begin{bmatrix} X_{norm}(t) \\ x_i(t) \\ x_j(t) \end{bmatrix}, \tag{3}$$

where $X_{norm} \in R^{N-2}$ is a vector containing all normal agents' states; $Q = I_{N-2} - \varepsilon(\bar{D} - \bar{A})$ and $B = \varepsilon\,[\,\mathbf{c}_i\ \mathbf{c}_j\,]$. Note that global consensus cannot be reached because of the existence of competitors. The result in our previous work (Zhao et al. (2014)) shows that, under the following assumptions:

- each normal agent has a path connecting to at least one competitor;
- $0 < \varepsilon < d_{\max}^{-1}$, where $d_{\max}$ is the largest out-degree of agents in the network,

the state of each normal agent will eventually reach a steady value, i.e., as $t \to \infty$, one has

$$X_{norm}(t) \to \bar{X} \triangleq (\bar{D} - \bar{A})^{-1}\,[\,\mathbf{c}_i\ \mathbf{c}_j\,] \begin{bmatrix} +1 \\ -1 \end{bmatrix}. \tag{4}$$

If $\bar{x}_k > 0$ ($\bar{x}_k < 0$), which means that agent k has a positive (negative) steady state, then agent k is considered as a supporter of competitor i (j). The case of $\bar{x}_k = 0$ implies that agent i is a neutral agent who does not support any competitor.

2.2 The Competitiveness Maximization Problem

In general, a competitor tends to possess greater influence than its rival so that it is more likely to win. Here, the influence difference between a competitor and its rival is referred to as *competitiveness* (more precise definition will be provided later), which can be used to represent the competition result. A natural way to increase one competitor's competitiveness is creating its new social links. An indicator vector $\phi \in R^{N-2}$ is defined as follows: if a new link is created from normal agent k to competitor i, $\phi_k = 1$; otherwise, $\phi_k = 0$. Then, after all new links being added from selected normal agents to competitor i,