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Antiferromagnetic geometric frustration under the influence of the next-nearest-neighbor interaction. An exactly solvable model

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HIGHLIGHTS

- Exact solution of antiferromagnetic model with NNN interaction on a recursive lattice is found.
- Full system of ground states of the model with exact values of magnetization is identified.
- Formation of additional plateau ground states related to NNN interaction is demonstrated.
- Exact expressions for all residual entropies are found and discussed.
- Role of single-point ground states in specific heat anomaly behavior is identified.

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ABSTRACT

The influence of the next-nearest-neighbor interaction on the properties of the geometrically frustrated antiferromagnetic systems is investigated in the framework of the exactly solvable antiferromagnetic spin-$1/2$ Ising model in the external magnetic field on the square-kagome recursive lattice, where the next-nearest-neighbor interaction is supposed between sites within each elementary square of the lattice. The thermodynamic properties of the model are investigated in detail and it is shown that the competition between the nearest-neighbor antiferromagnetic interaction and the next-nearest-neighbor ferromagnetic interaction changes properties of the single-point ground states but does not change the frustrated character of the basic model. On the other hand, the presence of the antiferromagnetic next-nearest-neighbor interaction leads to the enhancement of the frustration effects with the formation of additional plateau and single-point ground states at low temperatures. Exact expressions for magnetizations and residual entropies of all ground states of the model are found. It is shown that the model exhibits various ground states with the same value of magnetization but different macroscopic degeneracies as well as the ground states with different values of magnetization but the same value of the residual entropy. The specific heat capacity is investigated and it is shown that the model exhibits the Schottky-type anomaly behavior in the vicinity of each single-point ground state value of the magnetic field. The formation of the field-induced double-peak structure of the specific heat capacity at low temperatures is demonstrated and it is shown that its

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very existence is directly related to the presence of highly macroscopically degenerated single-point ground states in the model.

1. Introduction

The intriguing phenomenon of geometric frustration in magnetic systems [1] belongs, without doubt, among the most experimentally as well as theoretically investigated subjects in condensed matter physics (see, e.g., Refs. [2–10] and references cited therein). The reason for this interest is given especially by the fact that the presence of the geometric frustration in magnetic systems is the source of nontrivial peculiarities in their thermodynamical behavior such as the formation of magnetization plateaus in the low temperature limit, the existence of highly macroscopically degenerated ground states, i.e., the ground states with nonzero residual entropies, the appearance of various anomalies in the low temperature behavior of the specific heat capacity, and the observation of strong magnetocaloric effect in such systems. It is quite interesting that all of these properties of the frustrated magnetic systems, which are usually interpreted in the framework of the quantum nature of magnetism, can be seen and effectively described even on the classical level by using standard classical models of the statistical mechanics. Note that typical examples of magnetic geometrically frustrated systems are the antiferromagnetic systems on the lattices with triangular structure (the kagome lattice and the genuine triangular lattice), with tetrahedron structure (the pyrochlore lattice), or the well-known Shastry–Sutherland lattice.

From theoretical point of view, the most valuable models in the statistical mechanics are those which are exactly solvable simply because they represent a nontrivial theoretical background for potential fundamental understanding of physical properties of real macroscopic systems. However, on the other hand, it is also well-known that the set of exactly solvable models is rather restricted. Here, it is worth mentioning that even the classical models of the statistical mechanics, such as, e.g., the simple Ising model of the uniaxial magnetic systems, are exactly solvable at best on two-dimensional regular lattices and only in zero external fields (see, e.g., Refs. [11–13] and references cited therein) and, as for the three-dimensional regular systems, there exists no exact solution at all. At the same time, let us note that the importance of the presence of the external magnetic field in studied problems increases significantly especially when various magnetic frustrated systems are studied on regular lattices because the most interesting properties of such systems are revealed only under the influence of the external magnetic field. From this point of view, although the existence of exact solutions of the antiferromagnetic Ising model in zero external magnetic field on the two-dimensional regular lattices represents great achievement (see, e.g., Refs. [14,15], where the zero-field antiferromagnetic spin-$1/2$ Ising model on the triangular and kagome lattices were solved, respectively), nevertheless, they cannot give us any relevant information, e.g., about the system of all possible ground states and about their thermodynamical properties when the external magnetic field is applied on the studied magnetic systems.

Thus, it is evident that for a systematic theoretical investigation of even classical spin models in the presence of the external magnetic field it is necessary to use some kind of approximation. One of such approximation, which seems to be very effective especially for the investigation of basic thermodynamical properties of antiferromagnetic systems on various regular geometrically frustrated lattices, is to approximate a given regular lattice by the corresponding recursive lattice which takes into account basic geometrical properties of the regular lattice responsible for the frustration. Typical examples of such recursive lattices are, e.g., the so-called Husimi recursive lattices [16–18] which can be used for appropriate approximation of the standard regular lattices such as, e.g., the most studied kagome, triangular, or square lattice. Note that various thermodynamical properties of the classical statistical models on the recursive lattices were studied for a long time especially by using numerical investigation of the corresponding systems of recursion relations (see, e.g., Refs. [19–26] and references cited therein). There are at least two basic reasons for investigation of statistical models on various recursive lattices. The first reason is given by the fact that the obtained results on the recursive systems are usually in good accordance, at least qualitatively, with the properties of real physical systems. The second reason is that in this class of models there exist models which are solvable at an exact analytical level even in the presence of the nonzero external magnetic field [27–35].

Recently rather intensive interest has been devoted to the theoretical investigation of the regular two-dimensional so-called square-kagome lattice, which was introduced relatively recently in Ref. [36], and in literature is also known as squagome, squa-kagome, L4–L8 lattice, decorated square lattice, or shuriken lattice. The main reason for its theoretical investigation is related to the nontrivial structure of the elementary blocks from which the lattice is built (see the next section). Namely this fact allows one to perform systematic investigation of the influence of various additional interactions on the thermodynamical properties of the basic geometrically frustrated pure antiferromagnetic system. The properties of various classical as well as quantum magnetic systems on the square-kagome lattice were intensively studied recently (see, e.g., Refs. [36–49] and references cited therein). Here, it is also important to mention that the magnetic systems on the square-kagome lattice can be also interesting from phenomenological point of view because some real frustrated magnetic materials, e.g., gadolinium gallium garnet [50–53] with structure given by corner-sharing triangles, have very similar structure to the square-kagome lattice.

The properties of various classical antiferromagnetic geometrically frustrated systems with the square-kagome structure can also be investigated on the corresponding recursive lattice [49]. As we shall show in the present paper, the main