Adaptive fuzzy control for synchronization of coronary artery system with input nonlinearity

Zhanshan Zhao, Haoliang Cui, Jing Zhang and Jie Sun

***Abstract*—In this paper, we propose a parametric adaptive control strategy for synchronization of Takagi-Sugeno(T-S) fuzzy coronary artery system. We use the T-S fuzzy model to represent the coronary artery system because the coronary artery system has complicated nonlinear characteristic in reality. Based on the new model, a fuzzy parametric adaptive output feedback controller is designed to achieve the H_∞ synchronization of coronary artery system with input nonlinearity and parameter perturbations. Some simulation results are given to illustrate the effectiveness of our control strategy.**

***Index Terms*—Coronary artery system, Adaptive control, Fuzzy model, Input nonlinear, H_∞ synchronization.**

I. Introduction

CHAOS synchronization has been paid considerable attention among the scientists from biological engineering field such as epidemic diseases, nervous system and coronary artery system(CAS) [1, 2]. From the perspective of biology, CAS maintains our life by delivering oxygen and nutrition to myocardium. Once blood vessel of the coronary artery obstructed by thrombus, patients will suffering from a dangerous disease named myocardial infarction(MI). Therefore a lot of efforts have been done by the researchers among various areas. It's worth noting that Xu et al given the dynamics model of CAS in [3] which described CAS as a chaotic system:

$$\begin{aligned}\dot{x}_1(t) &= -bx_1(t) - cx_2(t),\\ \dot{x}_2(t) &= -(b+1)\omega x_1(t) - (c+1)\omega x_2(t) + \omega x_1^3(t)\\ &\quad +Ecos\sigma t \end{aligned} \tag{1}$$

where $x_1(t)$, $x_2(t)$ are the inner diameter and pressure changes of the coronary artery vessel, respectively. $Ecos\sigma t$ is used to describe the periodic perturbation.

Many existing works are based on the aforementioned model. In these researches, the treatment of MI are regarded as designing an appropriate control strategy to make the convulsionary vessel synchronize with a health one. In [4, 5], backstepping approach and nonlinear state feedback method are used to the CAS synchronization. Ref [6] utilizes sliding mode control method to achieve synchronization of CAS under the bounded uncertainties, takes full account of the presence of disturbances in the actual coronary artery system. Furthermore, the CAS synchronization in finite-time is achieved using high-order sliding mode adaptive control method in [7], makes the convulsionary vessel synchronize with a health one in finite-time,to ensure the control effect in the actual coronary artery system timeliness. Considering the time delay caused by medication time and drug absorption, a chaotic synchronization feedback controller with input time-varying delay is design to guarantee the control performance of CAS in [8]. The above articles are effective in considering the actual problems of CAS.

However, the CAS has complicated nonlinear characteristics. The nonlinear term in (1) will loss some information of the system. In the past two decades, T-S fuzzy model exhibited significant functions in approximating and describing complex nonlinear systems [9–14]. In this paper, we give a fuzzy CAS model which can retain much more information of nonlinear characteristics. Therefore, the study on CAS base on T-S fuzzy model compared to the previous research results is closer to the actual CAS.

Nonlinear effect widely exist in the natural phenomenon. The absorption and diffusion of drugs is also a nonlinear effect. Therefore, the medical efficacy is regarded as a nonlinear inputs in our paper.Compared to [6], our study is closer to the actual CAS. Furthermore, the parameters uncertainties are considered in drive-response systems so that the research has stronger robustness. Recently, adaptive fuzzy feedback control approach [15–18] is proven to be effective in nonlinear system control. Previous studies on CAS relied on deterministic mathematical models. However, the existing model is the approximation of CAS, there is a certain error. For this reason, we design a fuzzy adaptive controller, so that in the case of nonlinear input signal, the coefficient matrix exists for the modeling uncertainty, the response system and the drive system to achieve synchronization. In recent years, the researchers have proposed some new control strategies based on adaptive control and fuzzy control for different nonlinear systems, such as adaptive fuzzy control [19–21], observer-based fuzzy adaptive output-feedback control [22], adaptive tracking Control [23]. Sliding mode control [24] and adaptive control are the general control theory of chaotic synchronization, there are some methods to combine sliding mode control, such as adaptive sliding mode control [25], optimal guaranteed cost sliding mode control [26], adaptive fuzzy hierarchical sliding mode control [27].However, as we known, few researchers design control law based on fuzzy

Project supported by the National Natural Science Foundation of China (Grant Nos.61503280,61403278,61471243.) and the Science and Technology Commission of Tianjin Municipality (Grant No.15JCYBJC16100)

Zhanshan Zhao is with the School of Computer Science & Software Engineering, Tianjin Polytechnic University, Tianjin, 300387. E-mail:zhzhsh127@163.com

Haoliang Cui is with the School of Computer Science & Software Engineering, Tianjin Polytechnic University, Tianjin, 300387

Jing Zhang is with the School of Textiles, Tianjin Polytechnic University, Tianjin, 300387 and Tianjin Vocational Institute, Tianjin, 300410

Jie Sun is with the School of Computer Science & Software Engineering, Tianjin Polytechnic University, Tianjin, 300387

system which can better approach the real CAS with input nonlinearity and parameter perturbations.

Motivated by above discussions, we investigate the adaptive synchronization of CAS base on the T-S fuzzy model. An effective adaptive control strategy is proposed to the H_∞ synchronization of fuzzy CAS with the input nonlinear and parameter perturbations. The effectiveness of this strategy can be illustrated by the simulation in the following section.

A. Coronary artery fuzzy model

In this paper, we use uncertain T-S fuzzy model to describe CAS as follows:
Plant rule k: IF $\phi_1(t)$ is M_{k1}, $\phi_2(t)$ is $M_{k2}, \cdots, \phi_r(t)$ is M_{kr}, THEN

$$\begin{aligned} \dot{x}_m(t) &= (A_k+\Delta A_k)x_m(t)+(B_k+\Delta B_k)p(x_m(t),t) \\ &\quad +q(t)(k=1,\cdots,v) \\ y_m(t) &= Cx_m(t) \end{aligned} \tag{2}$$

where $\phi_j(t)(j=1\cdots r)$ is the premise variable. $M_{ij}(i=1\cdots k, j=1\cdots r)$ is the fuzzy set. r represents the number of the fuzzy rule, $x_m(t), y_m(t) \in \mathbb{R}^n$ are the state vector and output vector, respectively. $p(x_m(t),t)$ is the nonlinear term. $q(t)$ denotes a perturbation with certain period. $A_k, B_k, C \in \mathbb{R}^{n*n}$ are constant real matrices. $\Delta A_k, \Delta B_k \in \mathbb{R}^{n*n}$ represent the uncertainties of system which can be described as:

$$[\Delta A_k, \Delta B_k] = HF(t)[E_{ak}, E_{bk}] \tag{3}$$

where $H, E_{ak}, E_{bk} \in \mathbb{R}^n$ are known constant matrices and $F(t)$ is an unknown matrix function satisfying: $F^T(t)F(t) \leq I$.

Using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier,the dynamic fuzzy model in (2) can be represented by:

$$\begin{aligned} \dot{x}_m(t) &= \sum_{k=1}^{v} h_k(\phi(t))\{(A_k+\Delta A_k)x_m(t)+(B_k+\Delta B_k) \\ &\quad *p(x_m(t),t)+q(t)\} \\ y_m(t) &= Cx_m(t) \end{aligned} \tag{4}$$

where $h_k(\phi(t)) = \frac{\prod_{j=1}^{r} M_{kj}(\phi_j(t))}{\sum_{k=1}^{v}\prod_{j=1}^{r} M_{kj}(\phi_j(t))}(k=1,\cdots,v)$ is the normalized grade of membership and it satisfies: $\sum_{k=1}^{v} h_k(\phi(t)) = 1, h_k(\phi(t)) \geq 0$.

The fuzzy response system is given as follows:
Plant rule k: IF $\phi_1(t)$ is M_{k1}, $\phi_2(t)$ is $M_{k2}, \cdots, \phi_r(t)$ is M_{kr}, THEN

$$\begin{aligned} \dot{x}_s(t) &= (A_k+\Delta\tilde{A}_k(t))x_s(t)+(B_k+\Delta\tilde{B}_k(t))p(x_s(t),t) \\ &\quad +q(t)+d(t)+E\Omega(u(t))(k=1,\cdots,v) \\ &\quad y_s(t) = Cx_s(t) \end{aligned} \tag{5}$$

where $x_s(t), y_s(t) \in \mathbb{R}^n$ is the state vector and the output vector, respectively. $p(x_s(t),t)$ is the nonlinear term. $d(t)$ represents external disturbance. $E \in \mathbb{R}^{n*n}$ is constant real matrix. $\Delta\tilde{A}_k(t), \Delta\tilde{B}_k(t) \in \mathbb{R}^{n*n}$ denote the adaptive estimated value of $\Delta A_k, \Delta B_k$. Similar to (4), we infer the fuzzy response system (5) as:

$$\begin{aligned} \dot{x}_s(t) &= \sum_{k=1}^{v} h_k(\phi(t))\{(A_k+\Delta\tilde{A}_k(t))x_s(t)+(B_k+\Delta\tilde{B}_k(t)) \\ &\quad p(x_s(t),t)+q(t)+d(t)\}+E\Omega(u(t)) \\ y_s(t) &= Cx_s(t) \end{aligned} \tag{6}$$

Defining $e(t) = x_s(t) - x_m(t)$, the error system can be written as:

$$\begin{aligned} \dot{e}(t) &= \sum_{k=1}^{v} h_k(\phi(t))\{\Delta U_k x_s(t)+\Delta N_k p(x_s(t),t)\} \\ &\quad +\sum_{k=1}^{v} h_k(\phi(t))\{(A_k+\Delta A_k)e(t)+(B_k+\Delta B_k) \\ &\quad *p_e(t)+d(t)\}+E\Omega(u(t)) \end{aligned} \tag{7}$$

where

$$\Delta U_k = \Delta\tilde{A}_k(t) - \Delta A_k = (a_{kij})_{n\times n} \tag{8}$$
$$\Delta N_k = \Delta\tilde{B}_k(t) - \Delta B_k = (b_{kij})_{n\times n} \tag{9}$$
$$p_e(t) = p(x_s(t),t) - p(x_m(t),t) \tag{10}$$

The system (4) and (6) will be asymptotically synchronized if the synchronization error $e(t)$ satisfies $\lim_{t\to 0} e(t) = 0$. In this paper, we design an adaptive output feedback controller as follows:

$$u(t) = -\frac{\gamma(t)}{2}Ce(t) \tag{11}$$

where $u(t) = [u_1(t)\dots u_n(t)]^T \in \mathbb{R}^n$ is the control input vector, $\Omega(u(t)) = \sum_{k=1}^{v} h_k(\phi(t))[\omega_1(u_1(t))\dots\omega_n(u_n(t))]^T$ represents the nonlinear control input vector which satisfies the following inequality:

$$u_i(t)\omega_i(u_i(t)) \geq \nu_i(u_i(t))^2 \qquad (1 \leq i \leq m) \tag{12}$$
$$\nu^* = min\nu_i \tag{13}$$

ω_i is function, $\gamma(t)$ is an adaptive parameter and adjusted by the following adaptive law:

$$\dot{\gamma}(t) = \nu^*\delta\|Ce(t)\|^2, \gamma(0) > 0. \tag{14}$$

where δ and ν are positive parameters. By applying of the above adaptive controller, synchronization error $e(t)$ will converge to zero asymptotically. To obtain the synchronization conditions, the following lemma and assumptions will be used during the proof.

Lemma 1[28] For a symmetric matrix Z and appropriately dimensional matrices D, G and $F(t)$ satisfying $F^T(t)F(t) < I$. Inequality $Z + He\{DF(t)G\} < 0$ is true, if and only if the following inequality $Z + \varepsilon DD^T + \varepsilon^{-1}GG^T < 0$ holds for any $\varepsilon > 0$.

Assumption 1 The nonlinear function $p(x(t);t)$ satisfies the Lipschitz condition:

$$|p(x_m(t),t) - p(x_s(t),t)| \leq |L(x_m(t) - x_s(t))| \tag{15}$$

where L is the Lipschitz constant matrix.

Assumption 2 Matrix $P > 0$ and satisfies the following