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Reputation-based coevolution of link weights promotes cooperation in spatial prisoner's dilemma game

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ABSTRACT

Understanding and explaining the widespread cooperative behavior in society has become one open question. Many previous studies have been investigated this issue on static and unweight network, which is inconsistent with the realistic in sometimes. Motivated by this point, we consider a new setup about reputation-based dynamical weighted network. In particular, an individual with high reputation can receive a reward and its link weight will increase, while the individual with lowest reputation will be punished. Here, we introduce a parameter δ to control the range of link weight. Through numeric simulations, we find that coevolution setup can promote the evolution of cooperation. Particular, the larger value of δ, the higher level of cooperation. For the potential reason, we find it is related to the heterogeneous distribution of link weights.

1. Introduction

Cooperation among selfish and unrelated people is crucial for the development of human society. But according to natural selection, it is difficult to understand the emergence and maintenance of cooperative behavior [1–3]. In order to solve this issue, evolutionary game theory has proved to be a useful tool with competent theoretical framework in economy, biological systems and behavioral science [4–8]. In particular, the prisoner's dilemma game (PDG) is one of the most popular metaphor of social dilemma [9–16]. In the original one-shot PDG, two people decide the strategy cooperation (C) or defection (D) simultaneously without knowing what strategy the other chooses. Each people can receive the reward R or punishment P for mutual cooperation or defection. If one cooperates while the other defects, the former can receive the sucker's payoff P while defector can get the temptation T. The ranking of payoffs strictly followed $T > R > P > S$ and $2R > T + S$. The social dilemma can be expressed clearly: no matter what strategy opponent choose, defection always best for you. However, mutual cooperation can lead to highest total payoff.

Over the past decades, many setups have been proposed to resolve this dilemma, range from theory [17–20] to experiment [21–24]. Whereas, Nowak summarized five mechanisms: kin selection, direct reciprocity, indirect reciprocity, group selection and network reciprocity [25]. Among these achievements, network reciprocity has been proved to be the most effective way to promote the evolution of cooperation, which has attracted great interests to scholars from various disciplines. In spatial game, where players are arranged at structured network, cooperators can form compact clusters to avoid invade of defectors, and it has proven to be an effective way to promote cooperation [26]. Based on these mechanisms, many key factors have been discovered that can facilitate the evolution of cooperation include reputation [27–29], multigame [30,31], punishment [32–34], reward [35,36] and coevolution [37,38], to name but a few. Many results are achieved on static unweight networks. However, recent study also investigates how cooperation evolves when considering the weighted network [39]. Interestingly, the coevolution between strategies and network promote cooperative behavior effectively.

In fact, our living conditions are complex, and each individual may have different reputations. Obviously, we always keep a good relationship to the people with high credibility and away from low people. Sometimes, we even punish the people who have lowest reputation, and reward the individual with highest reputation. In line of this, this paper will propose a reputation-based method to study the coevolution of strategies and link weight on square lattice, we find some interesting phenomena and influential results.

The rest of this paper is composed of three sections. In Section 2, we present our evolutionary game model, including the new definition of reputation-based weight-change methods. Section 3 gives a description of numerical simulation results. Finally, we discuss the results and conclude the paper in Section 4.

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2. Model

We consider regular weighted network and Monte Carlo simulation on $L*L$ square lattice with period boundary. Each player is appointed to be either cooperator (C) or defector (D) with equal probability, which can be described as:

$$S_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad S_x = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

In our model, we choose weak prisoner's dilemma game, and the payoffs are $R=1$, $P=S=0$, $T=b>1$, these parameters strictly satisfy $T>R>P=S=0$. The payoff matrix can be given as:

$$M = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{2}$$

On structured population, the focal player x interact with one of its nearest eight neighbors y and obtain the payoff P_{xy}:

$$P_{xy} = s_x^T M s_y, \tag{3}$$

Next, considering the edge weight w_{xy} with payoff function, player x can get its cumulated fitness F_x:

$$F_x = \sum_{y \in \pi_x} w_{xy} P_{xy}, \tag{4}$$

where π_x represents the neighbors of player x. In reality, the relationship between A and B always different in their own opinion. Based on this, we use direction network which means $w_{xy} \neq w_{yx}$. Then, we introduce reputation effect in evolution of link weight. Thus, we define the reputation of individual x at step t as $R_x(t)$, if choose cooperation the reputation will plus 1, while defector's reputation will not change, which can be described as:

$$\begin{aligned} R_x(t) &= R_x(t-1) + 1, && x \text{ is cooperator}, \\ R_x(t) &= R_x(t-1), && x \text{ is defector}, \end{aligned} \tag{5}$$

Next, we will talk about the rule of update method. First, focal individual x and its neighbor y are randomly selected, and their reputations are calculated according to Eq. (5). In particular, if the reputation of player y is smallest in the eight neighbors of player x, the link weight x to y (w_{xy}) decrease β as the cost, and the link weight y to x (w_{yx}) will decrease β as a punishment. Similarly, if R_y is largest, w_{xy} decreases β, and w_{yx} will increase β as a reward. The updating can be described as:

$$\begin{cases} w_{xy} = w_{xy} - \beta \\ w_{yx} = w_{yx} - \beta \end{cases} \text{if } \mathrm{R_y} \text{ is the smallest}$$
$$\begin{cases} w_{xy} = w_{xy} - \beta \\ w_{yx} = w_{yx} + \beta \end{cases} \text{if } \mathrm{R_y} \text{ is the largest} \tag{6}$$

where $0<\beta<1$, and the value of link weight range from $1-\delta$ to $1+\delta$, where δ $(0<\delta<1)$ control the limit of weight. Then, fitness is calculated according to Eq. (4). If $F_x \geq F_y$, player x will not change its strategy. If $F_x < F_y$, player x updates the strategy from neighbor y with the probability:

$$\mathrm{w} = \frac{F_y - F_x}{\langle k \rangle D}, \tag{7}$$

where $<k>$ denotes the largest between the degree of player x and player y [34]. D denotes the maximal possible payoff difference for the prisoner's dilemma game. During a full Monte Carlo step all players update their strategies once on average. To worth raising, the key quantity the fraction of cooperation ρ_c was determined the last 1×10^3 steps of the full Monte Carlo simulations (MCS) with 1×10^5 steps, all simulations were carried out on lattices with $L=200$. Moreover, to avoid additional disturbances, the final results were averaged over up to 50 independent realizations for each set of parameter values in order to assure suitable accuracy.

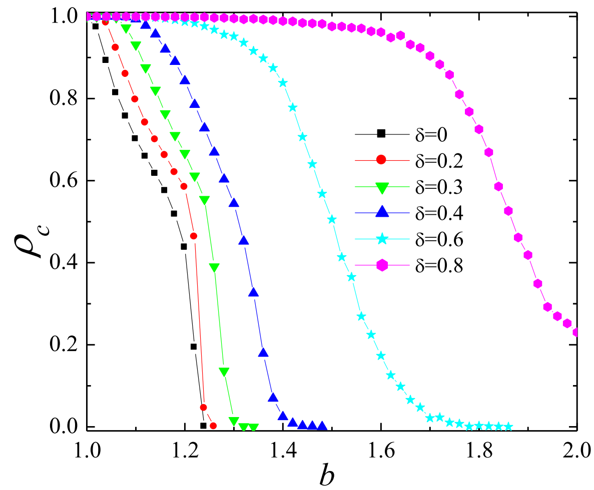

Fig. 1. The density of cooperators as function of temptation to defect b for different value of δ. For traditional version ($\delta = 0$), cooperators die out when b is small. With the increase of δ, cooperative behavior is promoted effectively. All the results are obtained for $\beta = 0.1$.

3. Results

First, we concentrate on how the frequency of cooperation depends on the temptation to defect b with different parameter δ (the limit of link weight) in Fig. 1. We find the coevolution of link weight and reputation influence the density of cooperation effectively. When $\delta = 0$, it returns back traditional model that each link weight is fixed at 1, and there is no much more information on direction links. So, the cooperation level down quickly with the increase of b, and die out even b is small. While considering evolution of link weights, the threshold b_c of cooperation vanishing become larger than traditional state. With the increase of parameter δ, cooperative behavior is promoted effectively. Attributing to link weight, cooperators can keep high level in the population with large δ. Especially, although large b can bring defectors more opportunities to exploit cooperators, defectors can't dominant the whole population even b reach to 2 when $\delta = 0.8$. It is worth pointing out that the larger the value of δ, the higher level cooperation can be observe in Fig. 1.

In order to explain above conclusion more detail, we provide the spatial pattern formed by two strategies for different value of parameter δ. Fig. 2 is obtained with fixed parameter $b=1.26$, $K=0.1$, $\beta=0.1$, cooperators and defectors are colored by red and yellow. For $\delta = 0$ (traditional version), defectors dominant square lattice in Fig. 2(a), cooperators have no chance surviving from defectors' invasion. When we consider evolution of link weight, cooperators can exist by forming compact clusters so that resist the invasion of defectors in Fig. 2(b). For $\delta = 0.4$ (Fig. 2(c)), larger and more compact cooperative clusters can protect more inner individuals, the distance among different cooperator domain is much smaller than their size, so they will outperform than defectors in stable state. Especially, when $\delta = 0.6$, in Fig. 2(d) cooperators nearly dominant whole population in grid lattice, defectors have no advantages to invade them. With the increase of δ, it is reveal that reputation-based coevolution of link weights can greatly promote cooperative behavior. But the reason why we can obtain above results, we will give a detail explanation in next paragraph.

Now that we testify the new setup can promote cooperation greatly, cooperators can form clusters resist the invasion of defectors. Furthermore, it is instructive to study the potential reason, and we do a research on the distribution of link weight for $b=1.26$, $\beta=0.1$ with different δ in Fig. 3. There exists an interesting re-