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Multi-parametric space-time *computational vademecum* for parametric studies: Application to real time welding simulations

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ABSTRACT

Real time simulations of welding processes remain intractable despite the impressive increasing computing power. This paper presents the case of a thermo-elasto-plastic problem with located moving heat loading. A novel non-intrusive *a posteriori* reduced order strategy for building multiparametric *computational vademecum* dedicated to real-time simulations of nonlinear thermo-mechanical problems is proposed. The high order proper generalized decomposition (HOPGD) is used to seek separated representation of solutions with some precomputed snapshots. Furthermore, a relaxation method is successfully applied to accelerate this procedure. The accuracy of the constructed *computational vademecum* is controlled by a localized multigrid selection method that allows an automatic selection of snapshots in the areas of interest of the parameter space. Examples of multiparametric *computational vademecum* taking into account some material parameters will be shown in this paper.

1. Introduction

In spite of the impressive progresses in computer science, traditional approaches reach some limitations when dealing with nonlinear parametric problems, like in inverse identification or optimization of the welding or additive manufacturing processes. A very large number of solutions of the concerned model has to be computed for different values of the problem parameters. In addition, when real-time simulations are required, it remains intractable with traditional computational approaches, due to the increasing degrees of freedom and complexity of the models.

In this context, model order reduction (MOR) techniques [1] have been developed recently. The resulting *computational vademecum* [2] (called also virtual charts [3,4] or meta-model computations), actually a series of parametric solutions, allows real-time simulations of parametric problems and open numerous possibilities in integrated simulation-based engineering. The construction of *computational vademecum* consists in usually two stages: offline and online. With the MOR techniques, the parametric solutions are computed and stored as a series of vectors (reduced bases) at offline stage once for all for building the *computational vademecum*. Then the *computational vademecum*, served as a sort of numerical or graphical handbook, can provide a real-time response for any value of parameters at the online phase. Depending on the way the reduced bases (RBs) are built with, two families of MOR approaches can be distinguished: *a posteriori* and *a priori*.

A posteriori approaches consist, usually, in employing the proper orthogonal decomposition (POD) method [5] to extract the most significant characteristic of the solution as RB that can be then applied to models with slight changes to the original one. Thus this kind of approaches needs some priori computed solutions and the resulting reduced order models can be usually solved efficiently. The POD-based MOR has been successfully applied in the context of solid mechanics (see e.g. Refs. [6–13]). The main issue of such approaches is the loss of efficiency when dealing with nonlinear problems with high parametric dependency. Several approaches, e.g. the Empirical Interpolation Method (and its discrete counterpart DEIM) [14,15], the hyper-reduction methods [9,16,17] and the asymptotic numerical method [5,18,19] that allows eliminating the re-computation of the tangent matrix, are introduced to accelerate the computations, but real time requirements remain intractable.

The other family of approaches, i.e. *a priori*, is based on the proper generalized decomposition (PGD) [20–22]. The main advantage of these approaches lies in the separated representations of solutions, which is firstly introduced under the name of “radial approximation” [23] by Pierre Ladeveze in the 80s within the framework of the LArge Time INcrements (LATIN) method [24,25], for solving high nonlin-

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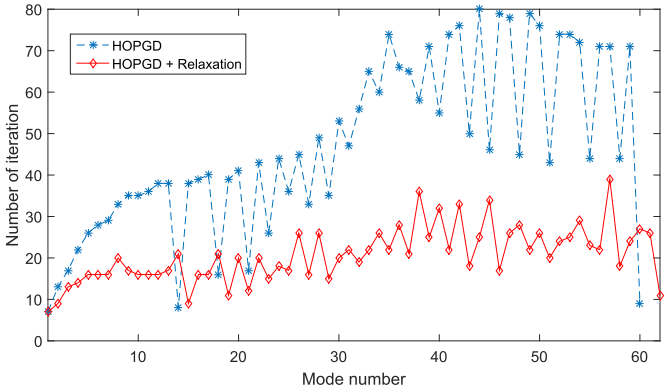

Fig. 1. Number of iterations during the construction of 3D PGD approximations.

Table 1
CPU time for 3D HOPGD.

	Number of modes	Iterations	CPU time
HOPGD	60	2832	400 s
HOPGD + Relaxation	62	1284	240 s

Table 2
CPU time of 5D HOPGD.

	Number of modes	Iterations	CPU time
HOPGD	91	4528	915 s
HOPGD + Relaxation	97	2641	563 s

ear problems. These PGD-based methods do not require any previous solution and the RBs are built on-the-fly by solving the resulting multi-dimensional models in which model parameters (boundary conditions, initial conditions, geometrical parameters, material and process parameters …) are considered as extra-coordinates. Many problems are solved using the PGD approaches; interested readers are referred to [20,21,26–29] and the references therein. A series of *computational vademecum* are constructed for different problems in sciences and engineering such as thermal control of industrial furnaces [30], shape optimization [2,4], computational surgery [31–33], etc. The use of *computational vademecum* can provide real-time responses at the online phase, since the multidimensional models have been solved offline for every possible value of parameters. This opens various possibilities and development in integrated simulation-based engineering. However, such approaches are usually intrusive. In particular, one can cite a recent paper [42] successfully using the PGD-based approach for the thermal process of welding, however, the application for nonlinear mechanical problems involving material plasticity is still under development.

For the sake of avoiding the intrusive aspect, recently, a POD-based *computational vademecum* [34] has been built through an *a posteriori* approach for welding processes. In this work, snapshots are computed with the standard finite element (FE) method. A manifold-based method [17,34,35] is proposed to interpolate both the space and time POD-RBs with respect to the variation of parameters. Thus there is no complex computation at the online phase (no equilibrium equations have to be solved) and real-time space-time responses can be obtained. This strategy of construction of *computational vademecum* without intrusive effects has been applied to standard welding simulations. The real-time *computational vademecum* can be helpful for engineers to make optimization decisions of welding processes.

In the same context, this paper presents an alternative *a posteriori* non-intrusive tool to build the multiparametric real time space-time *computational vademecum* using the high order PGD (HOPGD) method [36]. Similarly to PGD methods, the problem parameters are considered as extra-coordinates of solutions. Then a separated representation of solutions is constructed by HOPGD. However, the RB functions are computed offline, through an *a posteriori* approach, with some precomputed snapshots. The greedy algorithm incorporated with an alternative fixed point strategy is used for the search of the basis functions. In order to increase the convergence rate, the dynamic relaxation method (i.e. Aitken's Delta Squared method [37–39]) is proposed to accelerate the greedy algorithm. Its efficiency will be shown with several examples in this paper. Once the basis functions are constructed offline, solutions for new values of parameters can be provided online by the *computational vademecum* at real-time rates.

In order to control the accuracy of *computational vademecum*, a local refinement methodology [34] is applied to select the necessary snapshots in the parameter space for a given error. Exhaustive generations of snapshots, at expensive computational cost, can be then avoided.

Starting with the problem statement, this paper introduces the weakly coupled thermo-mechanical formulation in section 2. A new strategy with HOPGD method for building multiparametric *computational vademecum* is presented in section 3. The convergence accelerator of HOPGD will also be presented. Section 4 presents some application examples of the proposed approach. Finally, *computational vademecum* dedicated to parametric studies of welding processes will be shown at the end with a sensitivity analysis.

Notation: In this paper, the studied quantities of interest depend on the space-time coordinates $(\mathbf{X},t)\in\Omega\times[0,T]\subset\mathbb{R}^4$ and a set of parameters $\mu\in\mathcal{D}\subset\mathbb{R}^d$. The associated physical problem is then said to be a $(4+d)$D problem, in order to describe its dimension.

2. Problem statement

2.1. Strong formulation

Let us consider a transient thermo-elasto-plastic problem. Under weak coupling assumption, the transient heat transfer analysis can be carried out prior to the mechanical analysis, by solving the following governing equation

$$\rho C\frac{d\theta(\mathbf{X},t)}{dt}+\text{div }\mathbf{q}(\mathbf{X},t)=r(\mathbf{X},t)\quad\text{in the matrial domain }\Omega \tag{1}$$

where "div •" is the divergence operator with respect to the initial position $\mathbf{X}$, d•/dt the material time derivative, ρ and C are respectively the material density and specific heat capacity, and r is the internal heat generation.

The linear isotropic Fourier constitutive law is enforced here

$$\mathbf{q}(\mathbf{X},t)=-\mathbf{k}.\nabla\theta(\mathbf{X},t) \tag{2}$$

where $\mathbf{k}$ is the thermal conductivity.

Different boundary conditions (BCs) and initial conditions can be defined

$$\begin{cases}\mathbf{q}(\mathbf{X},t).\mathbf{n}(\mathbf{X},t)=\overline{\mathbf{q}}(\mathbf{X},t) & \text{on the surface }\partial\Omega^{\mathbf{q}}\\ \theta(\mathbf{X},t)=\overline{\theta}(\mathbf{X},t) & \text{on the surface }\partial\Omega^{\theta}\\ \theta(\mathbf{X},t=0)=0\end{cases} \tag{3}$$

Considering a reference frame $\widetilde{\Omega}$ moving at constant velocity $\mathbf{v}$, the original problem reads

$$\rho C\frac{\partial\theta(\mathbf{x},t)}{\partial t}+\mathbf{div}\ \mathbf{q}(\mathbf{x},t)+\rho C\mathbf{v}.\nabla\theta(\mathbf{x},t)=r(\mathbf{x},t)\quad\mathbf{in}\ \widetilde{\Omega} \tag{4}$$

or with the steady-state assumption, as proposed in Ref. [17]

$$\text{div }\mathbf{q}(\mathbf{x},t)+\rho C\mathbf{v}.\nabla\theta(\mathbf{x},t)=r(\mathbf{x},t)\quad\text{in }\widetilde{\Omega}\quad\text{with}\ \frac{\partial\theta(\mathbf{x},t)}{\partial t}=0 \tag{5}$$

where $\mathbf{x}$ is the current position vector of a material point defined by $\mathbf{X}$ at initial time in $\widetilde{\Omega}$. At each time t, $\mathbf{x}=\mathbf{X}+\mathbf{v}t$. Note that the steady-state