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Fractals or I.I.D.: Evidence of long-range dependence and heavy tailedness from modeling German equity market returns

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Abstract

Several studies find that the return volatility of stocks tends to exhibit long-range dependence, heavy tails, and clustering. Because stochastic processes with self-similarity possess long-range dependence and heavy tails, it has been suggested that self-similar processes be employed to capture these characteristics in return volatility modeling. In this paper, we find using high-frequency data that German stocks do exhibit these stylized facts. Using one of the typical self-similar processes, fractional stable noise, we empirically compare this process with several alternative distributional assumptions in either fractal form or I.I.D. form (i.e., normal distribution, fractional Gaussian noise, generalized extreme value distribution, generalized Pareto distribution, and stable distribution) for modeling German equity market volatility. The empirical results suggest that fractional stable noise dominates these alternative distributional assumptions both in in-sample modeling and out-of-sample forecasting. Our findings suggest that models based on fractional stable noise perform better than models based on the Gaussian random walk, the fractional Gaussian noise, and the non-Gaussian stable random walk.

JEL Classification: C41; G14

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1. Introduction

Because return volatility estimates are key inputs in valuation modeling and trading strategies, considerable research in the financial econometrics literature has been devoted to return volatility

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modeling. The preponderance of empirical evidence from financial markets throughout the world fails to support the hypothesis that returns follow a Gaussian random walk.[1] In addition to the empirical evidence, there are theoretical arguments that have been put forth for rejecting both the Gaussian assumption and the random walk assumption. One of the most compelling arguments against the Gaussian random walk assumption is that markets exhibit a fractal structure. That is, markets exhibit a geometrical structure with self-similarity when scaled (see Mandelbrot, 1963, 1997). As to this point, the normal distribution assumption and the random walk assumption cannot both be simultaneously valid for describing financial markets. It seems that the only way to explain fractal scaling is to abandon either the Gaussian hypothesis or the random walk hypothesis. By abandoning the Gaussian hypothesis, researchers end up with stable Paretian distributions.[2] The normal distribution is a special case with finite variance (details are discussed in Rachev & Mittnik, 2000). The implication of rejecting the random walk hypothesis is that researchers must accept that returns in financial markets are not independent but instead exhibit trends. Markets prone to trending have been characterized by long-range dependence and volatility clustering. Samorodnitsky and Taqqu (1994) demonstrate that the properties of some self-similar processes can be used to model financial markets that are characterized as being non-Gaussian and non-random walk. Such financial markets have been stylized by long-range dependence, volatility clustering, and heavy tailedness.

Long-range dependence or long memory denotes the property of a time series to exhibit persistent behavior, i.e., a significant dependence between very distant observations and a pole in the neighborhood of the zero frequency of its spectrum.[3] Long-range dependence time series typically exhibit self-similarity. The stochastic processes with self-similarity are invariant in distribution with respect to changes of time and space scale. The scaling coefficient or self-similarity index is a non-negative number denoted by H, the Hurst parameter. If $\{X(t+h) - X(h), t \in T\} \stackrel{d}{=} \{X(t) - X(0), t \in T\}$ for all $h \in T$, the real-valued process $\{X(t), t \in T\}$ has stationary increments. A succinct expression of self-similarity is $\{X(at), t \in T\} \stackrel{d}{=} \{a^H X(t), t \in T\}$. The process $\{X(t), t \in T\}$ is called *H-sssi* if it is self-similar with index H and has stationary increments (see Doukhan et al., 2003; Samorodnitsky & Taqqu, 1994).

In modeling return volatility, long-range dependence, volatility clustering, and heavy tailedness should be treated simultaneously in order to obtain more accurate predictions. Rachev and Mittnik (2000) note that for modeling financial data, not only does model structure play an important role, but distributional assumptions influence the modeling accuracy. A distribution that is rich enough to encompass those stylized facts exhibited in return data is the stable distribution. Fama (1963), Mittnik and Rachev (1993a, b), Mittnik et al. (2000, 2002), Rachev (2003), and Rachev et al.

[1] See Fama (1963, 1965), Mandelbrot (1963, 1997), and Rachev and Mittnik (2000).

[2] To distinguish between a Gaussian and non-Gaussian stable distribution, the latter is usually referred to as stable Paretian distribution or Lévy stable distribution. Referring to it as a stable Paretian distribution highlights the fact that the tails of the non-Gaussian stable density have Pareto power-type decay; referring to it as a Lévy stable distribution recognizes the pioneering works by Paul Lévy in characterizing the non-Gaussian stable laws (see Rachev & Mittnik, 2000).

[3] Baillie (1996) provides a survey of the major econometric research on long-range dependence processes, fractional integration, and applications in economics and finance. Doukhan et al. (2003) and Robinson (2003) provide a comprehensive review of the studies on long-range dependence. Bhansali and Kokoszaka (2006) review recent research on long-range dependence time series. Recent theoretical and empirical research on long-range dependence in economics and finance is provided by Rangarajan and Ding (2006) and Teyssiére and Kirman (2006). Sun et al. (2007) provide a review of long-range dependence research based on intra-daily data.