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A random dynamical system model of a stylized equity market

S.J. Hatjispyros, A.N. Yannacopoulos*

Department of Statistics and Actuarial Science, University of the Aegean, Greece

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Abstract

We propose a random dynamical systems model for a stylized equity market. The model generalises previous deterministic models for price formation in equity markets. We provide analytic results (existence of fixed points, existence of invariant measures) as well as numerical results indicating the dynamical richness of this simple model. The model can be used to assess the effects of uncertainty on the fundamentals on stock price dynamics.

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1. Introduction

The rôle of agents on the formation of prices in the stock market is a subject that has fascinated economists for a long time. There is a vast related literature on the subject to which thinkers of the caliber of John Maynard Keynes [1] or Fischer Black [2] have contributed. In recent years, models for the formation of prices using the theory of complex dynamical systems have been proposed. Richard Day [3–5] has proposed a dynamical systems model for an equity market. Eventhough the model was very simple (a one-dimensional map) it was designed in a very clever way so as to reflect well-established stylized facts for the stock market structure. As a result, the

*Corresponding author. Tel.: +30 22730 82335.
E-mail address: ayannaco@aegean.gr (A.N. Yannacopoulos).

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model could reproduce very well qualitative features of stock market prices such as transition between bull and bear regimes, etc. The model was purely deterministic.

The basic features of the model were the following. Each stock was supposed to somehow reflect the value of the fundamental it was representing. The stock market was assumed to consist of two types of investors, α-investors and β-investors and a mediator through which transactions were made. Each type of agent followed a different policy towards investment decisions. Type α-investors were those using sophisticated estimates of long run investment value. Such investors are those trying to buy when prices are below investment value (i.e., when chances of capital gain seem to be high) and trying to sell in the opposite case. Type β-investors cannot afford to be as sophisticated as α-investors. Such investors make investments decisions based on current investment value. In some sense, β-investors are the noise traders (to follow the terminology of Fisher Black) entering the market when prices are high and exiting the market in the opposite case. The third type of agent is the market maker (or mediator) who sets the price in response to excess demand or supply. The prices are supposed to be formed by Walrasian tattonement.

This simple dynamical model was found to present complicated behaviour giving rise to irregular oscillations of the prices (chaos). Furthermore, despite its simplicity it reproduced well-qualitative features of the prices, and could be used to understand the relative importance of the various types of investors in the market. As a result, it was greeted with great interest by the community (see the discussion following [5]).

The model of Day was a purely deterministic model. Eventhough there have been followups of Day's research (see e.g. Ref. [6] and references therein), all of them, at least to the best of our knowledge, have kept the deterministic nature of Day's original model. It is the aim of the present model to include randomness in Day's model. The randomness is introduced through the current investment value which is used by the β-investors in their investment decisions. The model can then be formulated as a random dynamical system. Through the use of recent advances of the theory of random dynamical systems, a qualitative and quantitative study of the evolution of prices in the stock market as a result of the interaction of different types of agents is possible. We see that the salient features and findings of the Day model (such as the existence of fixed points or the existence of invariant measures) are robust in the presence of random effects. Furthermore, the existence of unstable dynamics in the stock market, leading possibly to erratic oscillations, is shown through the calculation of Lyapunov exponents. The model allows us to monitor the effect of uncertainty of the fundamental to the stock prices and assess the effect of the underlying market dynamics on the propagation of uncertainty from the level of the fundamental to the level of the stock. The model, despite its simplicity, contributes to the interesting and important field of market microstructure, an active area of financial mathematics, the study of which is necessary for the understanding of the structure and function of financial markets.

The paper is organized as follows. In Section 2, we present the general features of our model and provide two special forms, a piecewise linear form and a piecewise monotone form. In Section 3, we present several qualitative and analytic results on the models, that is the existence of fixed points, the existence and approximation of