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Stable sets in matching problems with coalitional sovereignty and path dominance

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ABSTRACT

We study von Neumann Morgenstern stable sets for one-to-one matching problems under the assumption of coalitional sovereignty (C), meaning that a deviating coalition of players does not have the power to arrange the matches of agents outside the coalition. We study both the case of pairwise and coalitional deviations. We argue further that dominance has to be replaced by path dominance (P) along the lines of van Deemen (1991) and Page and Wooders (2009). This results in the pairwise CP vNM set in the case of pairwise deviations and the CP vNM set in the case of coalitional deviations. We obtain a unique prediction for both types of stable sets: the set of matchings that belong to the core.

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1. Introduction

In the one-to-one matching model known as the marriage problem, there are two disjoint sets of agents, say men and women. The problem is to match agents from one side of the market with agents from the other side, whereas each agent also has the possibility of remaining single. We refer to Roth and Sotomayor (1990) for a comprehensive overview on two-sided matching problems.

For marriage problems, stability is considered to be a central property. A matching is stable if each agent on one side is matched with an acceptable agent on the other side and no two agents of different sides would prefer to be matched to each other rather than to stick to their current situation. For marriage markets, this stability notion is known to be equivalent to core stability.

A matching is in the core if there is no subset of agents who, by forming only partnerships among themselves, can all obtain a strictly preferred outcome. Gale and Shapley (1962) show that the core of a marriage problem is non-empty. Although elements of the core have the property that they are stable once reached, it depends on the underlying environment whether it is possible to reach some core element from any initial situation. Stable sets as defined in von Neumann and Morgenstern (1944) address this concern.

A stable set is a set of outcomes that satisfies internal and external stability. As argued by von Neumann and Morgenstern (1944, p. 41), a stable set describes the "established order of society" or "accepted standard of behavior". Internal stability "expresses the fact that the standard of behavior is free from inner contradictions". External stability "can be used to discredit any non-conforming procedure".

vNM stable sets are crucially dependent on the concept of dominance. Under the standard definition, a matching is dominated by another matching if there is a coalition such that all its members prefer the latter matching to the former and no coalition member has a partner outside the coalition. A set of matchings is a vNM stable set if it satisfies the conditions of internal and external stability with respect to this dominance relation. Internal stability requires that no matching inside the set is dominated by a matching belonging to the set. External stability imposes that any matching outside the set is dominated by some matching belonging to the set. Ehlers (2007) shows that for one-to-one matching problems, the set of matchings in the core is a subset of any vNM stable set and a vNM stable set can contain matchings outside the core. Wako (2010) shows that the vNM stable set exists and is unique.

The standard dominance relation used to define vNM stable sets violates the assumption of coalitional sovereignty, the property that an objecting coalition cannot enforce the organization of

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agents outside the coalition. Such a violation is surprising since coalitional sovereignty is a very natural property to require when defining enforceability. For one-to-one matching problems, if a coalition deviates, then it is free to form any match between its members; it cannot affect existing matches between agents outside the coalition, and previous matches between coalition and non-coalition members are destroyed. The requirement that the coalition cannot affect existing matches between agents outside the coalition is violated in the standard definition of a vNM stable set.

Coalitional sovereignty is a natural requirement and has been imposed in other streams in the literature. In the literature on coalition formation, Hart and Kurz (1983) propose the γ and the δ model of coalition formation. Let some coalition structure be given and suppose some coalition deviates. Unaffected players are those players who are not part of the deviating coalition and were not together with any player of the deviating coalition in the original coalition structure. Coalitional sovereignty requires that nothing changes for the unaffected players and both the γ and the δ model respect coalitional sovereignty. Another issue is what happens to residual players, that is those players who were together with some player of the deviating coalition in the original coalition structure. The γ model assumes that the residual players become singletons after the deviation and the δ model assumes that they stay together. Kóczy and Lauwers (2004) study the accessibility of the core of a TU-game in coalitional form and also emphasize the importance of coalitional sovereignty. They call this property outsider independence.

For many-to-many matching problems, several authors have proposed and studied solution concepts that respect coalitional sovereignty, see in particular Echenique and Oviedo (2006) and Konishi and Ünver (2006). The important issue in many-to-many matching problems is not so much coalitional sovereignty, which is naturally assumed, but rather what happens to links between members of the deviating coalition and players outside that coalition, a problem closely related to the treatment of residual players in models of coalition formation.

An important stream in the literature on matching markets studies whether a decentralized process of successive blocking leads to a stable matching, see e.g. Roth and Vande Vate (1990), Klaus and Klijn (2007), and Kojima and Ünver (2008) for two-sided matching problems, and Chung (2000) and Diamantoudi et al. (2004) for roommate problems. All these papers formulate dominance relations that satisfy coalitional sovereignty.

Finally, several papers on the vNM stable set in the case of farsighted agents have used notions of enforceability that respect coalitional sovereignty, see Diamantoudi and Xue (2003) for hedonic games, Mauleon et al. (2011) for one-to-one matching problems, Klaus et al. (2011) for roommate markets, and Ray and Vohra (2015) for non-transferable utility games.

A further criticism of the standard definition of the vNM stable set is that it does not take into account that a deviation by a coalition can be followed by further deviations. This corresponds to the well-known critique by Harsanyi (1974) to the vNM stable set. Ray and Vohra (2015) emphasize that the notion of enforceability is especially delicate in the context of farsightedness as formulated by Harsanyi (1974). We argue here that, even in the standard myopic case, the same issue comes up when we apply the vNM stable set to one-to-one matching problems. We will follow the approach by van Deemen (1991) and Page and Wooders (2009), which takes into account that if a matching is blocked by some coalition and the resulting matching is not in the stable set itself, then further deviations will take place. This observation leads van Deemen (1991) to define the generalized stable set for abstract systems and Page and Wooders (2009) to define the stable set with respect to path dominance. We show by means of a simple example that not allowing for path dominance in the definition of the vNM stable set for one-to-one matching problems leads to highly undesirable conclusions when coalitional sovereignty is required.

Requiring coalitional sovereignty (C) and using the path dominance relation (P) to define internal and external stability, now referred to as CP internal stability and CP external stability, leads to the concept of the CP vNM set. Since in matching theory it is often assumed that only pairwise deviations are feasible, we also define the concept of the pairwise CP vNM set in an analogous way.

We show that there is a unique CP vNM set and a unique pairwise CP vNM set and that both sets coincide with the core. Although, as shown by Ehlers (2007), the core may not be a vNM stable set under the standard definition of the direct dominance relation,[1] it turns out to be the unique prediction when coalitional sovereignty and path dominance are taken into account.

Since the CP vNM set and the pairwise CP vNM set are based on paths of deviations resulting from the direct dominance relation, they should be thought of as myopic concepts. An alternative would be a farsighted approach based on the indirect dominance relation as introduced in Harsanyi (1974) and further developed in Chwe (1994). The vNM farsighted stable sets have been characterized in Mauleon et al. (2011) as the singleton core elements. Diamantoudi and Xue (2003) show that, in hedonic games with strict preferences, core partitions are farsightedly stable for a conservative notion of the concept.

The paper is organized as follows. Section 2 introduces one-to-one matching problems and standard notions of stability. Section 3 defines and characterizes both the CP vNM and the pairwise CP vNM set. Section 4 concludes and discusses directions for future research.

2. One-to-one matching problems

A one-to-one matching problem consists of a finite set of individuals N, partitioned into a set of men M and a set of women W. The set of non-empty subsets of N is denoted by $\mathcal{N}$. Each individual $i \in N$ has a complete and transitive *preference ordering* $\succ_i$ over the agents on the other side of the market and the prospect of being alone. Preferences are assumed to be strict. Let $\succ = ((\succ_m)_{m\in M}, (\succ_w)_{w\in W})$ be a preference profile. A one-to-one matching problem is a triple $(M, W, \succ)$.

A matching is a function $\mu : N \to N$ satisfying the following properties:

(i) For every $m \in M$, $\mu(m) \in W \cup \{m\}$.
(ii) For every $w \in W$, $\mu(w) \in M \cup \{w\}$.
(iii) For every $i \in N$, $\mu(\mu(i)) = i$.

The set of all matchings is denoted by $\mathcal{M}$. Given a matching $\mu \in \mathcal{M}$, individual $i \in N$ is said to be unmatched or single if $\mu(i) = i$. A matching μ is individually rational if each agent is acceptable to his or her mate, so for every $i \in N$ it holds that $\mu(i) \succ_i i$ or $\mu(i) = i$. A matching μ that is not individually rational can be blocked by any individual with an unacceptable partner. For a given matching μ, a pair $\{m, w\}$ is said to form a blocking pair if m and w are not matched to one another but prefer one another to their mates at μ, i.e. $w \succ_m \mu(m)$ and $m \succ_w \mu(w)$. A matching μ is stable if it is not blocked by any individual or any pair of agents.

For every $i \in N$, we extend the preference ordering $\succ_i$ over the agent's potential partners to the set of matchings in the following

[1] There are not so many classes of games where the core is the unique vNM stable set of the game. One example of such a class is the class of convex games, see Shapley (1971).

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