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Nontrivial Galilean-like invariance of the generalized higher-order nonlinear Schrödinger equation model with gravitation-like potential and the soliton analogies of the cosmic dark energy and antigravitation effects

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ABSTRACT

We reveal remarkable, but obviously only formal analogies and parallels between the soliton behavior in external gravitational-like potentials and the cosmic dark energy and antigravitation effects. The introduced generalized and exactly integrable higher-order nonlinear Schrödinger equation model for the wave function of a soliton in the attractive gravitational-like potential implies the existence of many fundamental nonlinear phenomena in completely different systems, independent of the physical origin of the nonlinearity and the higher-order linear and nonlinear dispersion. We show that the proposed model is governed by nontrivial Galilean-like invariance and, under the exact integrability conditions given by the Lax pair, demonstrates important formal nonlinear-optical analogies with enigmatic cosmic dark energy and antigravitation effects. Guided by interesting, but only formal analogy between the linear Doppler red shift and the nonlinear soliton self-frequency red shift effects, we reveal nontrivial features of the soliton propagation scenario in the attractive gravitation-like potential. Namely, the soliton attraction in the gravitation-like potential is changed to strong repulsion similar to the Newtonian particle dynamics in the repulsive antigravitational field. The wave-function red-shift effect (which is accumulated linearly with the propagation distance) is conserved under the transformation from attractive to repulsive scenarios in the ordinary space–time domain.

1. Introduction

The interpenetration of main ideas, analogies and theoretical methods being used in different fields of science and technology has become one of the decisive factors in the progress of science as a whole. The soliton paradigm as “e pluribus Unum” was proposed by Krumhansl as one of the possibilities to unite the science in which completely different phenomena from different areas are so often related by similar (and even common) mathematical algorithms [1]. Today, the soliton paradigm has brought together scientists from different areas of modern physics, including condensed matter and plasma physics, ultrafast photonics and nonlinear matter waves in Bose-Einstein condensates (BECs), fluid mechanics, theory of turbulence and phase transitions, elementary particle physics and nonlinear quantum field theory, biophysics, cosmology

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and nonlinear monster (rogue) waves in oceans [1–12]. Owing to their remarkable properties, solitons might appear both as the ideal data bit for high-bit-rates fiber-optics telecommunications [4–12] and the idealized mathematical structures for the description of extended "elementary" particles [13,14]. In particular, the concept of optical leptons and the soliton models of the hydrogen atom have been proposed recently [13–15]. All these facts make an impressive case in favor of the reality of Krumhansl's ideas [1]. It will suffice to mention that the impressive progress has been achieved in the successful search for the completely integrable extended mathematical models in different fields of nonlinear science (find, for example, the state of the art reviews in the recent publications and references therein [16–31]). Nonlinear waves and solitons have demonstrated themselves remarkably fruitful in different branches of science, and the most surprising thing is that, so far, the nonlinear-optical analogies and parallels have not been noticed in the so important cosmic dark energy and antigravitation problems.

Discovery of universal antigravitation arising at astronomical distances of the order of 5–8 billion light-years manifests itself as cosmic repulsion that acts between distant galaxies and overcomes their gravitational attraction giving rise to the accelerating expansion of the Universe. Previously unknown form of energy, that has been termed dark energy, dominates in the Universe at the present epoch and accounts for from 70% to 80% of the total mass and energy of the Universe. In macroscopic terms, it is a kind of a continuous medium that fills the entire Universe and is characterized by positive density and negative pressure (the details can be found in review papers [32–35]). Macroscopic quantum-mechanical methods and analogies, related to the superfluidity, superconductivity, and BECs, have been used in many theoretical works to describe dark energy and dark matter phenomena [36,37].

What other analogies and parallels are not yet established in this area?

This question is sending us in search for nonlinear-optical analogies connected, in particular, with the so-called nonlinear Euler–Heisenberg–Köckel vacuum caused by the virtual electron–positron pair production [38–40]. It should be emphasized that well-established fundamental physical mechanisms of the soliton formation, the scaling symmetries of the governing models, and the formal analogies between the macroscopic quantum-mechanical matter-wave (BECs) solitons, nonlinear-optical, and hydrodynamic nonlinear (ocean monster) waves, allow us to suggest that both the microscopic soliton models of extended quantum particles and "cosmic nonlinear matter-wave" spatial solitons with the sizes of galaxies and temporal solitons with the durations of million years are being formally permitted, even if the Euler–Heisenberg–Köckel nonlinearity is so very small.

We present here the generalized higher-order nonlinear Schrödinger equation (NLSE) model for the wave function of the soliton in the gravitational-like potential guided by interesting (but obviously only formal) analogy between the linear Doppler red shift and the nonlinear soliton self-frequency red shift effects [41–44].

We reveal that the soliton propagation scenario in the attractive gravitation-like potential is governed by nontrivial Galilean-like invariance, and under exact integrability conditions, demonstrates the appearance of unexpected strong repulsion. This effect is similar to the particle dynamics in the repulsive ("antigravitational") field. The second remarkable key feature of the solitonic analogy with the dark energy problem lies in the fact that the soliton spectrum is continuously shifted to the red frequency domain, so that the wave-function red-shift (which is accumulated linearly with the propagation distance) is conserved under the transformation from attractive to repulsive scenarios in the ordinary space–time domain.

2. Soliton frequency red-shift effect and the generalized higher-order nonlinear Schrödinger equation model with gravitation-like potential

On the basis of the well-known fact that in the nonrelativistic approximation, the nonlinear Klein-Gordon equation model can be transformed into the simplified model of the NLSE, we begin with the canonical "free" NLSE model

$$iQ_Z + \frac{1}{2}D_{20}Q_{TT} + R_{20}|Q|^2Q = 0. \tag{1}$$

When the following gauge transformation is applied to the NLSE model (1)

$$Q\,[T(x,t), Z(t)] = q(x,t)\exp\,[i\varphi(x,t)]\,, \tag{2}$$

$$\varphi(x,t) = 2\lambda_0 xt + \frac{2}{3}D_{20}\lambda_0^2 t^3, \quad T(x,t) = x + D_{20}\lambda_0 t^2, \quad Z(t) = t, \tag{3}$$

it creates the gravitational-like potential $U(x) = 2\lambda_0 x$ giving rise to the forced NLSE model

$$iq_t + \frac{1}{2}D_{20}q_{xx} + R_{20}|q|^2q - 2\lambda_0 xq = 0. \tag{4}$$

The linear (without self-action, $R_{20} = 0$) "predecessor" of this forced NLSE model (4) was known from the early days of quantum mechanics as the quantum model of the free fall of a particle over the Earth's surface in the gravitational potential $V(x) = mgx$ [45,46]

$$iq_t + \frac{1}{2}D_{20}q_{xx} - 2\lambda_0 xq = 0. \tag{5}$$